

# Long memory in return structures from developed markets

## Memoria larga en la estructura de los rendimientos en mercados desarrollados

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### **Abstract:**

*The present study aimed at investigating the existence of long memory properties in ten developed stock markets across the globe. When return series exhibit long memory, the series realizations are not independent over time and past returns can help predict future returns, thus violating the market efficiency hypothesis. It poses a serious challenge to the supporters of random walk behavior of the stock returns indicating a potentially predictable component in the series dynamics. We computed Hurst-Mandelbrot's Classical R/S statistic, Lo's statistic and semi parametric GPH statistic using spectral regression. The findings suggest existence of long memory in volatility and random walk for logarithmic return series in general for all the selected stock market indices. Findings are in line with the stylized facts of financial time series.*

### **Keywords:**

*Long memory, Rescaled range, Fractional integration, Spectral regression.*

### **Resumen:**

*El presente estudio pretende investigar la existencia de propiedades de memoria larga en diez mercados de valores de distintos países desarrollados. Cuando las series de rendimientos exhiben memoria larga, estas series no son independientes del tiempo y los rendimientos pasados pueden ayudar a predecir rendimientos futuros, violando por tanto la hipótesis de eficiencia de los mercados. Esto plantea un serio desafío a los que defienden que los rendimientos siguen un camino aleatorio, indicando un componente potencialmente predecible en la dinámica de las series. Hemos calculado el estadístico clásico de Hurst Mandelbrot (R/S), el estadístico de Lo y el estadístico semiparamétrico GPH utilizando un método de regresión espectral. Los resultados sugieren la existencia de memoria larga en la volatilidad de los rendimientos y un paseo aleatorio para los logaritmos de las series, en general para todos los índices de mercado seleccionados. Los resultados están en línea con hechos contrastados para series temporales financieras.*

### **Palabras claves:**

*Memoria larga, Rango reescalado, Integración fraccional, Regresión espectral*

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## 1. INTRODUCTION

The possible existence of long memory in stock market returns has imperative consequences for market efficiency and random walk behavior of the stock returns. The studies related to long range dependence include detection of long memory in the data, statistical estimation of parameters of long range dependence, limit theorems under long range dependence, simulation of long memory processes, and many others. Research on long memory processes were possibly stimulated by Hurst (1951) who quantified long memory using Rescaled Range analysis which considers the scaling behaviour of the range of partial sums of the variable under consideration. Studies on long memory processes in finance possibly originate from Mandelbrot (1971) suggesting that in the presence of long memory, pricing derivative securities with martingale methods may not be appropriate. Mandelbrot (1997) contains many of the early papers on the application of the Hurst exponent in financial time series. Since those days, the application of the long memory processes in economy has been extended from macroeconomics to finance. A good survey of the econometric approach to long-memory is given in Baillie (1996). Long-memory properties of financial time series indicates linear pricing models and statistical inferences about asset pricing models based on standard testing procedures may not be appropriate (Yajima, 1985). Several authors have claimed that the time series of stock returns for stock prices or indices display long-memory (Mandelbrot, 1971, Greene and Fielitz, 1977). However, Lo (1991) criticised the statistical R/S test used by Mandelbrot and Green and Fielitz on the ground that after accounting for short range dependence, it might yield a different result and proposed a modified R/S test statistic. However, Willinger et al. (1999) showed that the modified R/S test shows a bias towards rejection of long range dependence by rejecting the null hypothesis of short-memory when the degree of long-memory is not very high. Since financial data typically display low degree of long-memory, they claim that the result of Lo (1991) may not be conclusive.

It is well known in finance world that volatility is characterized by long memory. The consensus began to take shape with reports of hyperbolic decay in the autocorrelations of stock index volatilities (Ding et al., 1993) but gained momentum as fractionally-integrated GARCH models made inroads into the volatility modelling literature. There are many studies from developed markets showing that conditional volatility of returns on asset prices display long memory or long range dependence. Andersen and Bollerslev (1997; 1998), Ding, et al. (1993) and Breidt et al. (1998) find evidence of long-memory stochastic volatility in stock returns, Harvey (1993) finds evidence for this in exchange rates. Liow (2006) investigated persistence in international real estate market return and volatility on total-hedged and public real estate series. He finds little evidence of long memory for the return series, but overall long memory effect in volatility appears to be real and was less likely to be caused by shifts in variance for some Asia-Pacific real estate markets. These results led to the development of alternate models for volatility, such as Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) model. Harvey (1998) proposed an estimation method based on the spectral approximation to the Gaussian likelihood and the finite sample properties of this estimator were analyzed by Perez and Ruiz (2001). Granger and Hyung (2004) explained long memory phenomenon of asset returns by structural changes in GARCH and suggested that the time series with structural

breaks can induce a strong persistence in the autocorrelation function and hence generate spurious long memory. Banerjee and Urga (2005) provide a comprehensive survey of the literature on both long memory and structural breaks, features of which are almost observationally equivalent.

Presence of long memory properties in stock market returns still continues since empirical evidences reported in empirical studies is not strong enough but this fact has important consequences on the capital market theories. The presence of long memory dynamics cause nonlinear dependence in average asset returns. The primary implication of this circumstance is that return predictability is possible since an efficient market hypothesis is clearly rejected because stock market prices do not follow a random walk. It would also raise concern regarding linear modeling, forecasting, statistical testing of pricing models based on standard statistical methods, and theoretical and econometric modeling of asset pricing.

Possible existence of 'Taylor effect' is an interesting research area in finance. Taylor (1986) observed evidences of higher autocorrelations in absolute returns of assets than in squared returns. Ding et al. (1993) and Granger and Ding (1995, 1996) also found similar evidences and Granger and Ding (1995) referred this phenomenon as the 'Taylor effect.'

The present study aimed at investigating the existence of long memory properties in logarithmic return, absolute return and squared return series in ten developed stock markets across the globe. Logarithmic return (calculated at logarithmic first difference of the index values) is the most common form of return used in financial terminology. While 'squared return' is universally accepted and used as measure of volatility, 'absolute return' is also used as an alternative measure of volatility. Granger (1998) notes that long memory is usually discussed in the context of squared returns series, but that absolute returns series have more interesting statistical properties, thus motivating the investigation in this study. Absolute returns are robust in the presence of extreme or tail movements (Davidian & Carroll, 1987). Tail returns, with their generally accepted fat-tailed characteristic in financial time series, are of particular importance in market risk management and in associated risk measures such as value-at-risk and minimum capital requirements. Also absolute return modelling is more reliable than squared returns for the non-existence of a fourth moment commonly associated with financial returns (Mikosch and Starcia, 2000). There is a need for a more comprehensive study to make an attempt to find evidence of long memory or market inefficiency, more particularly, in the context of the emergence of new regulations, changing market micro structures in the developed markets. Moreover, it is also to be noted here that there remains always a natural need to vouch and verify the existing research findings. We have chosen ten leading indices in the ten chosen developed stock markets. The study also explores the existence of Taylor's effect in developed stock markets.

## 2. DEFINITION OF LONG MEMORY

The long memory describes the higher order correlation structure of a series. If a time series  $y_t$  is a long-memory process, there is persistent temporal dependence between observations widely separated in time. Such series exhibits hyperbolically decaying autocorrelations and low frequency distributions. If present, long memory has some serious signifi-

cance into the dynamics of the system; a shock in one point of time which leads to some increased risk and uncertainty in the market doesn't die down quickly if long memory is present. Rather, it stays on, although in a decaying fashion and affects future outcomes. Mathematically, if  $\lambda_s = \text{cov}(y_t, y_{t+s})$ ,  $s=0, \pm 1, \pm 2, \dots$ , and there exist constants  $k$  and  $\alpha$ ,  $\alpha \in (0,1)$  such that  $\lim_{s \rightarrow \infty} k \lambda_s s^{-\alpha} = 1$  then  $y_t$  is a long-memory process. A long memory process can be regarded as a fractionally integrated process, i.e., between stationary and unit root process. Like a stationary process, it is also a mean reverting process with a finite memory, i.e., it comes back to equilibrium after experiencing a shock. But unlike an autoregressive stationary process, it shows a much slower hyperbolic rate of decay rather than exponential, and the process takes much larger time to adjust back to equilibrium. When a time series have unit root at level but its first-differences are stationary, it is said to be I(1) process (integrated of order one). A stationary process is said to be I(0) process (integrated of order zero). Using the same notation, long memory process is I(d) process, where  $d$  lies between 0 and 1, i.e., a fraction. In the frequency domain, long memory financial time series have typical spectral power concentration near zero or at low frequencies and then it is declining exponentially and smoothly as the frequency increases (Granger, 1966). Long memory has also been called the "Joseph Effect" by Mandelbrot and Wallis (1968), a biblical reference to the Old Testament prophet who foretold of the seven years of plenty followed by the seven years of scarcity that Egypt was to experience. This in general parlance indicates that good times beget good times and bad times beget bad.

### 3. METHODOLOGY FOR TESTING LONG-MEMORY PROCESSES

The empirical determination of the long-memory property of a time series is a difficult since strong autocorrelation of long-memory processes makes statistical fluctuations very large. Thus tests for long-memory tend to require large quantities of data. In this paper we tested the stationary properties of all the data series using Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) test. We have tried to capture the long memory property of financial data using classical rescaled-range (R/S) analysis (Hurst, 1951; Mandelbrot, 1972), modified rescaled-range (R/S) analysis introduced by Lo (1991) and the spectral regression method suggested by Geweke and Porter-Hudak (1983). The above tests were applied on logarithmic return series, absolute return series and squared return series. The referred methods are detailed below.

#### 3.1. Rescaled-range (R/S) analysis

R/S analysis provides a measure of long range dependence based on the evaluation of the Hurst's exponent of stationary time series introduced by English hydrologist H.E. Hurst in 1951. The Hurst exponent was built on Einstein's contributions regarding Brownian motion of physical particles and is frequently used to detect long memory in time series. R/S analysis in economics was introduced by Mandelbrot (1971, 1972, 1997) who argued that this methodology was superior to the autocorrelation, the variance analysis and to the spectral analysis. Let  $X(t)$  be the price of a stock on a time  $t$  and  $r(t)$  be the logarithmic

return denoted by  $r(t) = \ln\left(\frac{X_{t+1}}{X_t}\right)$ . The R/S statistic is the range of partial sums of deviations of times series from its mean, rescaled by its standard deviation. Hence, if  $r(1), r(2), \dots, r(n)$  denotes logarithmic asset returns and  $\bar{r}_n$  represents the mean return given by  $\bar{r}_n = \frac{1}{n} \sum_{t=1}^n r(t)$ , where ‘n’ is the time span considered, the rescaled range statistic is given by  $\left(\frac{R}{S}\right)_n = \frac{1}{\sigma_n} \left[ \max_{1 \leq k \leq n} \sum_{t=1}^k (r(t) - \bar{r}_n) - \min_{1 \leq k \leq n} \sum_{t=1}^k (r(t) - \bar{r}_n) \right]$  where  $\sigma_n$  is the maximum

likelihood estimate of simple standard deviation:  $\sigma_n = \frac{1}{n} \sum_{t=1}^n (r(t) - \bar{r}_n)^2$ . The first term in the bracket is the maximum of the partial sum of the first k deviations of r(t) from the sample mean, which is nonnegative. The second term in the bracket is the corresponding minimum of the partial sums, which is nonpositive. The difference of these two quantities, called “range” is always nonnegative and makes the rescaled range,  $\left(\frac{R}{S}\right)_n \geq 0$ . The

advantage of the classical R/S analysis is that the results are reliable regardless whether the distribution of the series is Gaussian or not. The null hypothesis of the test is that there is no long-range dependence in the series. This test is performed by calculating the confidence intervals with respect to generally accepted significance level, and to see whether the rescaled range statistic lies in or outside the desired interval. The critical values for the above two tests are given in Lo, 1991, table II.

A drawback of the R/S analysis is that its measure of long range dependence is affected by short range dependence that may be presented in the financial data. Hence we consider estimating modified R/S statistic proposed by Lo (1991).

### 3.2. Modified rescaled-range (R/S) analysis

We conducted the modified R/S analysis suggested by Lo (1991) for long memory that examines the null hypothesis of no long range dependence at different significance levels. Lo’s modified version of the statistic takes account of short-range dependence by performing a Newey-West correction (using a Bartlett window) to derive a consistent estimate of the long-range variance of the time series. Lo’s modified R/S statistic, denoted by  $Q_n$  is defined as:

$$Q_n = \frac{1}{\sigma_n(q)} \left[ \max_{1 \leq k \leq n} \sum_{t=1}^k (r(t) - \bar{r}_n) - \min_{1 \leq k \leq n} \sum_{t=1}^k (r(t) - \bar{r}_n) \right]$$

where  $\sigma_n^2(q)$  is the Newey-West (1987) estimate of long run variance of the series defined as:

$$\sigma_n^2(q) = \frac{1}{n} \sum_{t=1}^n (r(t) - \bar{r}_n)^2 + 2 \sum_{j=1}^q \omega_j(q) \gamma_j$$

defined as: where  $\gamma_j$  represents the sample autocovariance of order  $j$ , and  $\omega_j(q)$  represents the weights applied to the sample autocovariance at lag  $j$  ( $1, 2, \dots, q$ ).  $\omega_j(q)$  are defined as the following Barlett weights:

$$\omega_j(q) = 1 - \frac{j}{q+1}$$

The second term in the long run variance equation intended to capture the short term dependence. The lag length  $q$  used to estimate the heteroskedasticity and autocorrelation corrected (HAC) standard deviation is extremely crucial for modified R/S test of long memory. We have used bandwidth selection procedures suggested by Andrew (1991) to find the lag length.

### 3.3. The Spectral Regression Method

A stationary long memory process can be characterized by the behaviour of the spectral density  $f(\lambda)$  function which takes the form  $f(\lambda): c |1 - e^{-i\lambda}|^{-2d}$ , as  $\lambda \rightarrow 0$  with  $d \neq 0$ ,  $c \neq 0$ ,  $d$  is the long memory parameter (or fractional differencing parameter) and  $0 < |d| < 0.5$ . In order to estimate the fractional differencing estimator  $d$ , Geweke and Porter-Hudak (1983) proposed a semi-parametric method of the long memory parameter  $d$  which can capture the slope of the sample spectral density through a simple OLS regression based on the periodogram, as follows:  $\log I(\lambda) = \beta_0 - d \log \{4 \sin^2(\lambda_j / 2)\} + v_j$ ,  $j=1, \dots, M$ ; where  $I(\lambda)$  is the  $j^{\text{th}}$  periodogram point;  $\lambda_j = 2\pi j / T$ ;  $T$  is the number of observations;  $\beta_0$  is a constant; and  $v_j$  is an error term, asymptotically i.i.d, across harmonic frequencies with zero mean and variance known to be equal to  $\pi^2 / 6$ .  $M = g(T) = T^\mu$  with  $0 < \mu < 1$  is the number of Fourier frequencies included in the spectral regression and is an increasing function of  $T$ . As argued by GPH the inclusion of improper periodogram ordinates  $M$ , causes bias in the regression which result in an imprecise value of  $d$ . To achieve the optimal choice of  $T$ , several choices are established in terms of the bandwidth parameter  $M = T^{0.45}$ ;  $T^{0.50}$ ; ...,  $T^{0.7}$ . The GPH fractional differencing test is performed on the stock return aiming at a prospective gain in estimation efficiency. The fractional distinction test tends to find out fractal constitution in a time series based on spectral investigation of its low-frequency dynamics.

## 4. DATA

The series studied in this analysis include ten stock market indices, AEX (Netherlands), ^AORD (Australia), DAX (Germany), DJA (USA), FCHI (France), FTSE 100 (UK), HANGSENG (Hongkong), NIKKEI (Japan), NZE 50 (New Zealand) and STRAITS TIMES (Singapore) at daily frequencies. The market classification as developed is based on Morgan Stanley Capital International (MSCI). The MSCI market classification scheme depends on the following three criteria: economic development, size and liquidity, and market accessibility. A market is classified as developed if: i) the country's Gross National Income per capita is 25% above the World Bank high income threshold for 3 consecutive years; ii) there is a minimum number of companies satisfying minimum size and liquidity requirements; and iii) there is a high openness to foreign ownership, ease of capital

inflows/outflows, high efficiency of the operational framework and stability of the institutional framework. The period of study is from January 2005 to July 2011. The daily closing values of the individual indices were taken and daily logarithmic index returns were calculated using the relation  $r(t) = \ln(p_{t+1}) - \ln(p_t)$  where  $r(t)$  is the return on the index on  $t$ -th day,  $\ln(p_{t+1})$ ,  $\ln(p_t)$  represents natural logarithm of index value on  $t+1$  day and  $t^{\text{th}}$  day respectively. We test for long memory on logarithmic return, absolute return (mod value) and squared return series from the stock markets referred above.

## 5. FINDINGS

### 5.1. Descriptive Statistics

The statistical summaries of logarithmic return, absolute return and squared return series of all the indices are reported in **Table 1** which shows that average logarithmic return of seven indices ^AORD, DAX, DJA, FTSE 100, HANGSENG, NZX 50, and STRAITS TIMES are positive. The logarithmic return series of seven indices are negatively skewed while other three are positively skewed and all ten return series are leptokurtic. This along with high value of Jarque-Bera statistic clearly suggests that logarithmic return series of both the indices cannot be regarded as normally distributed. The absolute return series and squared return series of all the ten indices are positively skewed and leptokurtic indicating non normal distribution.

Table 1  
Descriptive Statistics

Indices	Data	Mean	Median	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
AEX	RET	-0.00003	0.00055	0.014796	-0.159232	12.53456	6336.518
	SQR	0.00022	0.00004	0.000743	8.245191	84.72715	483981.5
	ABS	0.00954	0.00618	0.011309	3.43741	20.35517	24261.88
^AORD	RET	0.00007	0.00055	0.011951	-0.55286	8.137586	1902.147
	SQR	0.00014	0.00004	0.000381	9.038697	126.8836	1079542
	ABS	0.00841	0.00618	0.008491	2.617565	14.66688	11262.62
DAX	RET	0.000311	0.001026	0.014209	0.135695	11.564	5105.45
	SQR	0.000202	0.0000426	0.000656	10.4208	148.5377	1503184
	ABS	0.009551	0.006528	0.010523	3.256275	21.04484	25593.4
DJA	RET	0.00015	0.00075	0.013479	-0.143218	10.73939	4108.639
	SQR	0.00018	0.00004	0.000567	9.299241	122.1793	996647.9
	ABS	0.00893	0.00612	0.0101	3.060672	17.93027	17836.29

Indices	Data	Mean	Median	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
FCHI	RET	-0.00002	0.00028	0.014939	0.139911	11.14367	4625.705
	SQR	0.00022	0.00005	0.000711	9.032366	104.5622	741337.5
	ABS	0.01005	0.00685	0.011051	3.228661	19.81492	22602.54
FTSE 100	RET	0.000114	0.000535	0.013306	-0.111072	11.467420	4926.60
	SQR	0.000177	0.000038	0.000573	9.070070	107.669800	774892.40
	ABS	0.008873	0.006156	0.009914	3.286173	19.980070	22764.28
HANGSENG	RET	0.000264	0.000607	0.017637	0.085279	12.02245	5561.239
	SQR	0.000311	0.000058	0.001033	10.60141	151.658	1539892
	ABS	0.011561	0.007613	0.01332	3.215278	20.72255	24273.66
NIKKEI	RET	-0.00009	0.00048	0.01683	-0.57684	12.17072	5695.538
	SQR	0.00028	0.00006	0.00095	10.60768	147.82870	1428364
	ABS	0.01131	0.00780	0.01246	3.38563	22.64538	28786.06
NZX 50	RET	0.00006	0.00044	0.00790	-0.31660	7.71956	1540.965
	SQR	0.00006	0.00002	0.00016	10.51878	169.53740	1914881
	ABS	0.00569	0.00413	0.00548	2.51779	15.22086	11872.77
STRAITS TIMES	RET	0.000244	0.000621	0.013286	-0.346848	9.498384	2923.87
	SQR	0.000176	0.0000371	0.000514	8.272209	98.35018	641137.9
	ABS	0.008918	0.006089	0.009849	2.841394	15.48525	12882.18

RET – Logarithmic Return Series, SQR – Squared Return Series, ABS – Absolute Return Series.

## 5.2. Unit Root tests

The results of unit root tests are displayed in **Table 2**. The null hypothesis of presence of unit root in ADF test and PP test is rejected at 1% level of significance for logarithmic return, absolute return and squared return series of all ten indices indicating all the data series are stationary.



Table 2  
Unit Root Tests

Indices	Data	ADF	PP	Indices	Data	ADF	PP
AEX	RET	-42.0436 ***	-42.0436 ***	^AORD	RET	-42.0522***	-42.0522***
	SQR	-4.4416 ***	-85.5997***		SQR	-7.0459***	-48.3395 ***
	ABS	-5.2382 ***	-69.4831***		ABS	-6.5539***	-50.2863 ***
DAX	RET	-42.0477***	-42.0477 ***	DJA	RET	-33.2366 ***	-44.3978 ***
	SQR	-4.4151 ***	-106.7454 ***		SQR	-4.8819 ***	-82.3488 ***
	ABS	-5.5997 ***	-80.7019 ***		ABS	-5.3254 ***	-66.6536***
FCHI	RET	-43.7130 ***	-43.7130***	FTSE 100	RET	-19.2083***	-43.6156***
	SQR	-4.5899 ***	-87.9003 ***		SQR	-4.4409 ***	-80.1036 ***
	ABS	-5.9249***	-65.5307 ***		ABS	-7.6952 ***	-42.6195 ***
HANGSENG	RET	-42.1530 ***	-42.1530***	NIKKEI	RET	-41.5425 ***	-41.5425 ***
	SQR	-6.8983 ***	-32.5338 ***		SQR	-6.6884 ***	-38.6122 ***
	ABS	-5.2128 ***	-59.0043 ***		ABS	-7.7521***	-37.5714 ***
NZX 50	RET	-37.9154 ***	-37.9154 ***	STRAITS TIMES	RET	-41.0268 ***	-41.0268 ***
	SQR	-5.1796 ***	-73.1594 ***		SQR	-6.3265***	-63.0542 ***
	ABS	-5.6547 ***	-60.0368***		ABS	-5.7316***	-65.8550 ***

a) The critical values are those of Mackinnon (1991).

b) \*\*\* represent the rejection of null hypothesis at 1% level of significance.

### 5.3. Visual Interpretation: Autocorrelation Function (ACF)

The ACF was plotted against the time lag for logarithmic return, absolute return and squared return series of all the ten indices. The lag was taken upto 36 days. The autocorrelation is found to decay quickly and is insignificant in the logarithmic return series of all the indices. However in case of absolute and squared return series, a slow decay in autocorrelation is observed. The ACF of the data series (**Figure 1**) indicates short memory in logarithmic return but long range dependence or persistence for absolute and squared return series in developed stock markets. The findings also support existence of Taylor Effect in the selected developed markets as autocorrelations of absolute returns (**Figure 2**) are usually larger than those of squared returns (**Figure 3**).

Figure 1

Visual Interpretation: Autocorrelation Function (ACF) of logarithmic return series of ten developed stock indices

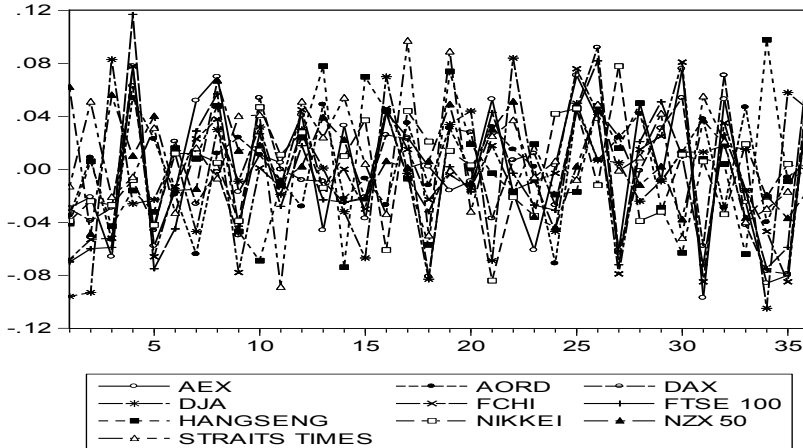


Figure 2

Visual Interpretation: Autocorrelation Function (ACF) of absolute return series of ten developed stock indices

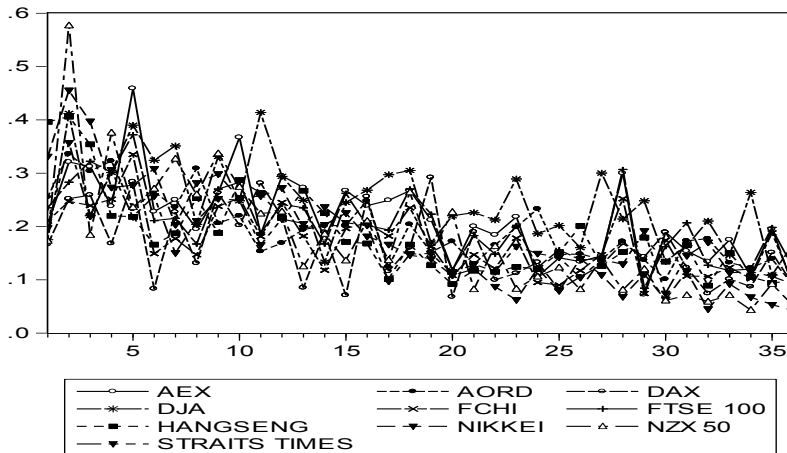
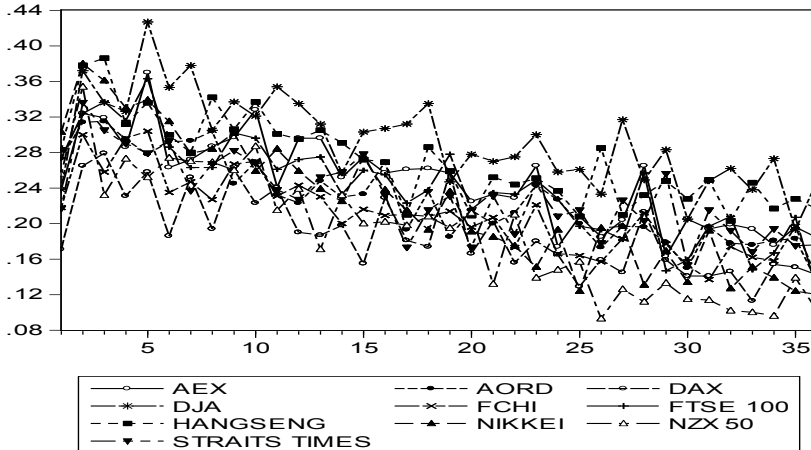


Figure 3

**Visual Interpretation: Autocorrelation Function (ACF) of squared return series of ten developed stock indices**



**5.4. Rescaled-Range (R/S) Analysis: Hurst-Mandelbrot’s Classical R/S Statistic and Lo Statistic**

The results of Rescaled-Range (R/S) Analysis are presented in **Table 3** where Hurst-Mandelbrot’s Classical R/S Statistic and Lo Statistic are displayed. The estimated values of Hurst-Mandelbrot’s Classical R/S Statistic suggest that the null hypothesis of no long-range dependence in case of logarithmic return series of all ten indices could not be rejected at a generally acceptable level of significance as estimated values of the statistic fall within the acceptance region. However, for both absolute and squared return, the null hypothesis is rejected at 1% level of significance. The critical values of the statistic are obtained from Lo (table II, 1991). This clearly indicates that although logarithmic returns may not have long memory, returns without signs as well as volatility as measured by squared returns shows existence of long run dependence in the series. We also computed Lo’s statistic since Classical R/S Statistic is sensitive to short range dependence and may give biased results in the case of short-range dependence and heterogeneities. The Lo statistic displayed in **Table 3** also shows that the null hypothesis of no long-range dependence in case of logarithmic return series of all ten indices could not be rejected at a generally acceptable level of significance as estimated value of the statistic falls within the acceptance region. For absolute return series, Lo statistic rejects the null hypothesis at 1% level of significance for all the ten indices and findings are similar in case of squared returns as well. The results of both the tests are consistent and indicate short memory for logarithmic return series and long memory for volatility in general for the selected developed stock markets.

Table 3

**Hurst-Mandelbrot's Classical R/S Statistic and Lo Statistic**

Indices	Data	Hurst-Mandelbrot's Classical R/S Statistic	Lo Statistic	Indices	Data	Hurst-Mandelbrot's Classical R/S Statistic	Lo Statistic
AEX	RET	1.69	1.69	^AORD	RET	1.67	1.67
	SQR	5.22	3.02		SQR	5.5	2.92
	ABS	6.96	3.8		ABS	6.97	3.66
DAX	RET	1.6	1.6	DJA	RET	1.52	1.52
	SQR	4.73	3.14		SQR	5.55	3.37
	ABS	6.14	3.98		ABS	7.11	3.72
FCHI	RET	1.45	1.45	FTSE 100	RET	1.3	1.3
	SQR	4.64	2.85		SQR	5.08	2.83
	ABS	6.36	3.61		ABS	6.81	3.51
HANGSENG	RET	1.57	1.57	NIKKEI	RET	1.36	1.36
	SQR	5.56	2.64		SQR	4.32	2.09
	ABS	8.26	4.15		ABS	5.73	2.77
NZX 50	RET	1.91	1.85	STRAITS TIMES	RET	1.97	1.97
	SQR	5.19	3.07		SQR	5.79	3.47
	ABS	6.43	3.45		ABS	7.56	4.2

Note: Critical values:

10% level of significance	[0.861, 1.747]
5% level of significance	[0.809, 1.862]
1% level of significance	[0.721, 2.098]

**5.5. The Spectral Regression Method (GPH statistic)**

**Table 4** report estimates of the fractional differencing parameter ( $d$ ) for the daily logarithmic return, absolute return and squared return series of all ten indices from ten developed stock markets. The test examine the null hypothesis of short memory ( $H_0 : d = 0$ ) against long memory alternatives ( $H_1 : d \neq 0$ ) for a range of bandwidth ( $M = T^{0.45}, T^{0.50}, \dots, T^{0.7}$ ). The estimates of  $d$  are statistically significant for all ten indices in absolute and square return series. The null hypothesis of short memory is rejected and the findings show that long memory exists in absolute return and volatility in the selected stock markets. However the findings are mixed in case of logarithmic return series. Estimate of  $d$  is found to be statistically significant at two chosen bandwidths in case of Netherlands(AEX) whereas it is

found significant at one of the chosen bandwidth in case of Australia(^AORD), New Zealand (NZX 50) and Singapore (STRAITS TIMES). The null of short memory in logarithmic return series is rejected in case of Germany, USA, France, UK, HongKong and Japan.

Table 4

**GPH estimate of fractional differencing parameter (d)**

Indices	Data	M=T <sup>0.45</sup>	M=T <sup>0.50</sup>	M=T <sup>0.55</sup>	M=T <sup>0.60</sup>	M=T <sup>0.65</sup>	M=T <sup>0.70</sup>
AEX	RET	0.1995 (0.1372) [1.4539]	0.2519** (0.1075) [2.3445]	0.1538 (0.0902) [1.7036]	0.1597 (0.0856) [1.8663]	0.1428** (0.0684) [2.0880]	0.0569 (0.0540) [1.0538]
	SQR	0.5523*** (0.0714) [7.7339]	0.6039*** (0.0614) [9.8341]	0.6348*** (0.05445) [11.6593]	0.7610*** (0.0702) [10.8285]	0.5406*** (0.0571) [9.4552]	0.5244 *** (0.0457) [11.4562]
	ABS	0.6534*** (0.1306) [5.0041]	0.6198*** (0.0930) [6.6641]	0.7218*** (0.0940) [7.6754]	0.6640*** (0.0738) [8.9890]	0.5377*** (0.0612) [8.7781]	0.4414*** (0.0495) [8.9155]
^AORD	RET	0.1867 (0.1213) [1.5401]	0.2523** (0.1002) [2.5188]	0.0868 (0.0850) [1.0211]	0.0687 (0.0773) [0.8883]	0.0167 (0.0599) [0.2802]	0.0149 (0.0488) [0.3064]
	SQR	0.4680*** (0.0880) [5.3157]	0.5477*** (0.0735) [7.4510]	0.4886*** (0.0697) [7.0069]	0.4849*** (0.0555) [8.7232]	0.4756 *** (0.0452) [10.5136]	0.5199*** (0.0429) [12.0966]
	ABS	0.5197*** (0.1014) [5.1265]	0.6487*** (0.1154) [5.6206]	0.5735*** (0.0956) [5.9970]	0.5455*** (0.0727) [7.4974]	0.5512 *** (0.0658) [8.3682]	0.4811*** (0.0534) [9.0009]
DAX	RET	0.2677 (0.1544) [1.7338]	0.1425 (0.1122) [1.2707]	-0.0130 (0.0907) [-0.1437]	-0.0297 (0.0729) [-0.4080]	0.0184 (0.0630) [0.2919]	0.0061 (0.054) [0.1135]
	SQR	0.5136*** (0.0702) [7.3067]	0.6811*** (0.0913) [7.4541]	0.5549*** (0.0712) [7.7889]	0.6034*** (0.0638) [9.4530]	0.4845*** (0.0539) [8.9818]	0.3410*** (0.0465) [7.3293]
	ABS	0.5787*** (0.1534) [3.7729]	0.6394*** (0.1074) [5.9530]	0.5559*** (0.0912) [6.0907]	0.5524*** (0.0821) [6.7267]	0.4850*** (0.0633) [7.6514]	0.3484*** (0.0502) [6.9321]
DJA	RET	0.1482 (0.1404) [1.0559]	0.1050 (0.1019) [1.0313]	-0.0171 (0.0858) [-0.1994]	-0.0306 (0.0650) [-0.4708]	0.0175 (0.0581) [0.3022]	-0.0283 (0.0486) [-0.5825]
	SQR	0.7270*** (0.1513) [4.8059]	0.8562 *** (0.109) [7.8550]	0.6717*** (0.0839) [8.0037]	0.7128*** (0.0661) [10.7850]	0.6746 *** (0.0519) [12.9878]	0.5333*** (0.0430) [12.3837]
	ABS	0.6627*** (0.1003) [6.6076]	0.7844*** (0.0961) [8.1604]	0.7262 *** (0.0911) [7.9673]	0.7168*** (0.0672) [10.6575]	0.6586 *** (0.0547) [12.0386]	0.5916*** (0.0468) [12.6399]

Indices	Data	$M=T^{0.45}$	$M=T^{0.50}$	$M=T^{0.55}$	$M=T^{0.60}$	$M=T^{0.65}$	$M=T^{0.70}$
FCHI	RET	0.1027 (0.1328) [0.7737]	0.1965 (0.1059) [1.8552]	0.0448 (0.0901) [0.4973]	-0.0009 (0.0740) [-0.0013]	0.0102 (0.0638) [0.1605]	-0.0062 (0.0508) [-0.1226]
	SQR	0.6037*** (0.1078) [5.6003]	0.6099*** (0.0863) [7.0631]	0.5536*** (0.0688) [8.0391]	0.5943*** (0.0684) [8.6808]	0.4418 *** (0.0541) [8.1570]	0.4085*** (0.0458) [8.9181]
	ABS	0.5637*** (0.1097) [5.1376]	0.5779*** (0.0818) [7.0591]	0.5987 *** (0.0815) [7.3403]	0.5962*** (0.0782) [7.6223]	0.5395*** (0.063) [8.5632]	0.4229*** (0.0502) [8.4111]
FTSE 100	RET	0.0217 (0.1521) [0.1431]	0.1173 (0.1136) [1.0323]	-0.0569 (0.0888) [-0.6407]	-0.0618 (0.0727) [-0.8498]	0.0116 (0.0630) [0.1853]	-0.0160 (0.0508) [-0.3158 ]
	SQR	0.5387*** (0.0706) [7.6249]	0.6574*** (0.0679) [9.6799]	0.5648 *** (0.0652) [8.6548]	0.5966*** (0.0681) [8.7590]	0.4454*** (0.0522) [8.5178]	0.4845*** (0.0441) [10.9650]
	ABS	0.5808*** (0.1167) [4.9778]	0.6638*** (0.0895) [7.4120]	0.6223*** (0.0800) [7.7766]	0.5704*** (0.0695) [8.2060]	0.5440*** (0.0581) [9.3610]	0.4328*** (0.0451) [9.5770]
HANGSENG	RET	0.1088 (0.1229) [0.8860]	0.3056 (0.1614) [1.8948]	0.1285 (0.1185) [1.0845]	0.0293 (0.0883) [0.3323]	-0.0404 (0.0649) [-0.6232]	0.0163 (0.0545) [0.3001]
	SQR	0.3922*** (0.0657) [5.9675]	0.5608*** (0.0970) [5.7813]	0.4852*** (0.0766) [6.3297]	0.5235*** (0.0602) [8.6909]	0.3993*** (0.0512) [7.7887]	0.3010*** (0.0399) [7.5371]
	ABS	0.5989*** (0.1071) [5.5905]	0.6568*** (0.0913) [7.1951]	0.5893 *** (0.0800) [7.3649]	0.6105*** (0.0630) [9.6895]	0.5202*** (0.0528) [9.8531]	0.4342*** (0.0460) [9.4282]
NIKKEI	RET	0.1359 (0.1442) [0.9429]	0.1358 (0.1071) [1.2684]	0.0331 (0.0820) [0.4039]	0.0679 (0.0657) [1.0331]	0.0413 (0.0559) [0.7388]	0.0088 (0.0480) [0.1839]
	SQR	0.3143*** (0.0581) [5.4108]	0.4148*** (0.0664) [6.2409]	0.4719*** (0.0575) [8.2066]	0.6120*** (0.0581) [10.5340]	0.5143*** (0.0518) [9.9168]	0.4568*** (0.0427) [10.6899]
	ABS	0.5208*** (0.0955) [5.4520]	0.5242*** (0.0762) [6.8756 ]	0.5949*** (0.0816) [7.2909]	0.6369*** (0.0739) [8.6151]	0.5575*** (0.0604) [9.2232]	0.5164*** (0.0480) [10.7447]
NZX 50	RET	0.0018 (0.1202) [0.0152]	0.1654 (0.0976) [1.6937]	0.1089 (0.0825) [1.3198]	0.1207 (0.0703) [1.7165]	0.1299** (0.0590) [2.2012]	0.0750 (0.0460) [1.6307]
	SQR	0.3001*** (0.0528) [5.6850]	0.3726*** (0.0414) [8.9823]	0.4841*** (0.0500) [9.6742]	0.5531*** (0.0456) [12.1230]	0.6195*** (0.0420) [14.7465]	0.5686*** (0.0453) [12.5491]
	ABS	0.4555*** (0.1193) [3.8175]	0.5075*** (0.0925) [5.4830]	0.6070*** (0.0878) [6.9084]	0.5674*** (0.0655) [8.6519]	0.5749*** (0.0559) [10.2715]	0.4598*** (0.0494) [9.2928]

Indices	Data	$M=T^{0.45}$	$M=T^{0.50}$	$M=T^{0.55}$	$M=T^{0.60}$	$M=T^{0.65}$	$M=T^{0.70}$
STRAITS TIMES	RET	0.1831 (0.1404) [1.3041]	0.2477** (0.1046) [2.3694]	0.1655 (0.0833) [1.9863]	0.0729 (0.0635) [1.1470]	0.0599 (0.0615) [0.9733]	0.0769 (0.0506) [1.5183]
	SQR	0.4827*** (0.0707) [6.8197]	0.5487*** (0.0785) [6.9904]	0.5138*** (0.06664) [7.7113]	0.5846*** (0.0609) [9.5965]	0.5000*** (0.0583) [8.5636]	0.4641*** (0.0500) [9.2727]
	ABS	0.6988*** (0.1872) [3.7325]	0.6692 *** (0.1381) [4.8477]	0.5019*** (0.1014) [0.9502]	0.4883*** (0.0778) [6.2719]	0.4268*** (0.0629) [6.7844]	0.3868 *** (0.0490) [7.8908]

a) \*\*\*, \*\* and \* represents the rejection of null hypothesis at 1%, 5% and 10% level of significance respectively.  
 b) Standard errors in ( ) and t-statistics in [ ].

## 6. CONCLUSION

Efficient market hypothesis in its weak form suggests that asset prices reflect all available information and asset prices should fluctuate as random white noise which is satisfied by unpredictable behaviour of asset returns. In presence of long memory, market efficiency hypothesis is violated since return series are not independent over time and therefore past returns may be used to predict future returns. Exploring long memory property is appealing for derivative market participants, risk managers and asset allocation decisions makers, whose interest is to reasonably forecast stock market movements. The study examined the evidence of long memory in the ten developed markets – 4 from Europe, 5 from Pacific and the US. To test the presence of long-memory in asset returns, we computed Hurst-Mandelbrot’s Classical R/S statistic, Lo’s statistic and semi parametric GPH statistic. All the tests both are consistent with long range dependence in the absolute return and squared return series. Findings largely support the Taylor effect as autocorrelations of absolute returns are usually larger than those of squared returns and the estimate of the fractional differencing parameter is generally higher for the absolute returns than that of squared returns. Overall findings did not suggest long-term memory in chosen stock market logarithmic returns indicating developed stock market returns follows a random walk. Absence of long memory in logarithmic return series of the indices show no evidence against the weak form of market efficiency in stock returns. Our findings are consistent across all the selected developed countries where stock market of all countries show long range dependence in the absolute return and squared return series but did not suggest long-term memory in logarithmic returns. Absence of long memory will imply that assets are not systematically over or under-valued, which provides justification for passive index investment. Investors may expect a normal (risk adjusted) rate of return while firms should expect to receive a fair value for securities that they sell. Apparent past price patterns are not predictive for future prices leaving little scope for profitable arbitrage opportunities. Presence of long memory in squared returns indicates volatility of asset returns can be modeled using returns from the recent as well as remote past and hence derivative instruments can now be more efficiently priced. Another important implication concerning the existence of long memory in asset returns series is concerns the application of risk analysis models to estimate potential losses, which is the case of Value at Risk (VaR). In this respect,

identifying the presence of long memory in financial assets series must aid in producing more conservative and precise estimations in VaR analysis. Also the relevance of linear pricing models and statistical inferences about asset pricing models based on standard testing procedures is not questionable in absence of long range dependence in stock returns. Given the financial economic environment, settlement cycles, strong regulatory authority and market micro structure in the developed markets, a possible explanation for absence of long memory in return series may be based on the grounds that developed markets are informationally efficient, prices tend to reflect all publicly available information and any new information is fully arbitrated away. An alternative explanation was suggested by Lo(1991) when he suggested that "... we find little evidence of long-term memory in historical U.S. stock market returns. If the source of serial correlation is lagged adjustment to new information, the absence of strong dependence in stock returns should not be surprising from an economic standpoint, given the frequency with which financial asset markets clear. Surely financial security prices must be immune to persistent informational asymmetries, especially over longer time spans". The financial market regulators in these developed markets may look into the sources of long memory in volatility of stock returns to improve efficiency levels.

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