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Abstract

When the in-plane bias magnetic field acting on a flat circular magnetic dot is smaller than the saturation field, there are two stable competing magnetization configurations of the dot: the vortex and the quasi-uniform (C-state). We measured microwave absorption properties in an array of non-interacting permalloy dots in the frequency range 1–8 GHz when the in-plane bias magnetic field was varied in the region of the dot magnetization state bi-stability. We found that the microwave absorption properties in the vortex and quasi-uniform stable states are substantially different, so that switching between these states in a fixed bias field can be used for the development of reconfigurable microwave magnetic materials.

Keywords: ferromagnetic resonance, magnetic vortex, dynamical susceptibility, magnetic dot, microwaves

Regular two-dimensional arrays of ferromagnetic nanodots magnetized to saturation can be used for the development of novel magnetic microwave materials, where the spectrum of microwave absorption is mainly determined by the nanodot sizes and interdot distances [1, 2]. In particular, dense arrays of interacting magnetic dots (or pillars) can be used for the development of microwave absorption materials reconfigurable by the application of pulsed bias magnetic fields [3].

When a flat magnetic nanodot is magnetized by an in-plane bias magnetic field that is smaller than the field of magnetic saturation, there co-exist several competing minima of the dot magnetic energy (stable states) as a function of the dot magnetization configuration. One of the minima corresponds to the dot ground state and others correspond to metastable states. For instance, the dot could be in either quasi-uniform or vortex magnetization stable states [4]. In our current work, we are going to show that the arrays of flat, non-interacting nanodots magnetized by sufficiently small in-plane bias fields, so that the state of individual dots is bi-stable, could be used for the development of reconfigurable magnetic microwave materials in which the microwave absorption properties change drastically, when the stable state of magnetic dots is changed even at the same magnitude of the applied bias magnetic field. Many types of field-tuned linear and nonreciprocal microwave devices, such as delay lines, frequency filters, valves etc, can be created on the basis of 2D nanodot arrays [5]. Recently, it has been demonstrated that, using the patterned magnetic film media [2], it is also possible to develop useful nonlinear microwave devices, which were impossible to create using continuous films due to parametric excitations of spin waves.

Although it is intuitively clear that the microwave absorption properties of a magnetic nanodot should change when the stable state of static magnetization in it is changed, to the best of our knowledge, the microwave absorption in magnetic nanodot arrays existing in the interval of bias fields where the magnetization state of a nanodot is bi-stable, has not been studied experimentally so far. The sensitivity of nanoparticle arrays to external microwave magnetic fields with frequency $f > 1$ GHz is an important parameter for the applications of these arrays in real microwave devices. This sensitivity, characterized by the dynamical magnetic susceptibility [5], has not been explored in the bias field interval, where dots can exist in both quasi-uniform and vortex stable states. The experimental study of the microwave susceptibility of a magnetic dot array in a wide frequency range and in this interval of relatively low (compared to the field of magnetic saturation) magnetic fields is the subject of our present article.

To investigate experimentally the microwave absorption in a two-dimensional array of in-plane magnetized non-interacting magnetic dots, we used a standard technique of vector network analyzer ferromagnetic resonance (VNA-FMR) [6–14], which has been employed previously to investigate the microwave properties of magnetic dot arrays [8, 9, 11–15]. The microwave measurements were conducted by the VNA set-up ZVA-8 (Rohde&Schwarz). A permalloy ($Ni_{80}Fe_{20}$ alloy) film patterned into a two-dimensional regular array of flat circular magnetic dots was placed over the central line of a coplanar waveguide, so that the microwave magnetic field of the coplanar waveguide was oriented perpendicular to the direction of the in-plane bias magnetic field. The applied in-plane bias field was quite low (<600 Oe), to operate in the region of the system bi-stability existing between the vortex nucleation and annihilation fields, when both the vortex state and quasi-uniform state are stable and competing. The coplanar Cu line was sputtered on a laminate wafer of thickness 0.9 mm and dielectric permittivity $\varepsilon = 3.66$, the width of the central line was 450 μm, and the gaps were 70 μm. The investigated dot array (with a patterned area of 5×5 mm^2) was composed of circular permalloy
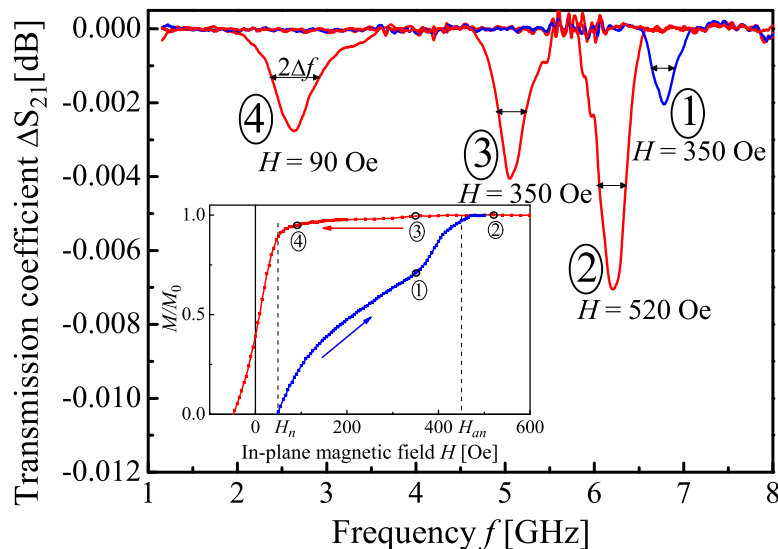
Figure 1. The experimental microwave absorption lines of a square array of cylindrical permaloy dots (dot radius $R = 150$ nm, dot thickness $L = 14$ nm, and with an edge-to-edge interdot distance of 300 nm) measured at four different points ((1)–(4)) on the static hysteresis loop of the array shown in the inset. The marked fields H_n, H_{an} are the vortex nucleation and annihilation fields, respectively. Points (2), (3) and (4) ($H = 520$, 350, 90 Oe) are situated on the upper branch of the hysteresis loop corresponding to the quasi-uniform stable state of the dots, while point (1) ($H = 350$ Oe) is situated at the lower branch of the hysteresis loop corresponding to the vortex stable state of the dots. Note that the absorption curves corresponding to points (1) and (3) are taken at the same magnitude of the in-plane bias magnetic field $H = 350$ Oe. The parameters of the measured absorption curves: resonance frequency f_{res}, half-linewidth Δf and resonance susceptibility $\chi''(f_{res})$ as functions of the bias magnetic field are presented in figure 2.

dots with a diameter $2R = 300$ nm and thickness $L = 14$ nm. The edge-to-edge interdot distance was 300 nm, which guarantees an almost complete absence of interdot magnetostatic interactions.

The first measurement performed on this dot array was the measurement of a static hysteresis loop performed using a SQUID-magnetometer (see the inset in figure 1). The loop has a two-lobe shape typical of the vortex state dots [4], with a vortex nucleation field of $H_n \approx 50$ Oe and a vortex annihilation field of $H_{an} = 450$ Oe (see the lower lobe in the inset of figure 1). The dots are saturated ($M = M_0$) at a field above the vortex annihilation field H_{an}. When the bias magnetic field is decreasing and reaches the value below H_{an} (see the upper lobe in the inset of figure 1), the magnetization first decreases slightly, keeping a quasi-uniform ground state in the dot, and then decreases drastically due to the dot transition, first into a deformed C-state (just above the nucleation field H_n [15]) and, eventually, to the vortex ground state (just below H_n [4]). The vortex state is stable up to $H = -H_{an}$, when the bias field decreases to negative values.

The microwave transmission coefficient through the coplanar line containing the studied dot array

$$S_{21}^2(f, H) = \frac{P_{\text{out}}(f, H)}{P_{\text{in}}} \tag{1}$$

(defined as the ratio of the output power P_{out} to the input power P_{in}, which was kept constant at the level of 1 mW at all frequencies) was measured as a function of the input frequency f within the frequency range of 1–8 GHz at different points situated at both lower (vortex) and upper (quasi-uniform) lobes of the static hysteresis loop at different values of the bias magnetic field H. Since the absolute value of the microwave absorption in the dot array is rather small, to accurately characterize this absorption we measured, at each value of the bias magnetic field H, the ratio of the transmission coefficient at a given value of H and the transmission coefficient in the absence of the bias magnetic field:

$$\Delta S_{21}^2(f, H) = S_{21}^2(f, 0) / S_{21}^2(f, H). \tag{2}$$

The graphs of this quantity in dB $\left(\Delta S_{21} \text{dB} = 10 \log(\Delta S_{21}^2) \right)$ as a function of the input frequency f (taken at several points situated on the hysteresis loop) of the dot array are presented in figure 1. Each plot represents the resonance absorption line with the field-dependent resonance frequency $f_{\text{res}}(H)$. We denote the resonance transmission coefficient as $\Delta S_{21}^2\left(f_{\text{res}}, H\right)$. The essential parameters of these microwave absorption curves (the resonance frequency f_{res}, the linewidth Δf, defined as the full width at half maximum (FWHM), and the value of the transmission coefficient at resonance $\Delta S_{12}^2(f_{\text{res}})$) were also measured at both the upper (quasi-uniform) and lower (vortex) lobes of the hysteresis loop.

To quantitatively characterize the microwave absorption in the magnetic dot array, it is convenient to introduce the notion of the imaginary part of the microwave magnetic susceptibility $\chi''(f, H)$ [2], which is proportional (with a constant coefficient C) to the ratio of the power P_{abs} absorbed by the dot array to the incident power P_{in}:

$$\chi''(f, H) = C \frac{P_{\text{abs}}(f, H)}{P_{\text{in}}}. \tag{3}$$

Using the measured values of the resonance transmission coefficients at different bias fields H, $\Delta S_{21}^2(f_{\text{res}}, H)$, we calculated the values of the microwave resonance magnetic susceptibility of the magnetic dot array normalized by the resonance susceptibility value in the quasi-uniform ground state, at the frequency $f_s = 4$ GHz situated in the middle of the frequency interval of our measurements:

$$\chi_n''\left(f_{\text{res}}, H\right) = \frac{\Delta S_{21}^2\left(f_{\text{res}}, H\right)}{\Delta S_{21}^2\left(f_s, H\right)} l\left(f_{\text{res}}\right), \tag{4}$$

where $l(f)$ is a coefficient accounting for the frequency dependence of the losses $L_{\text{CPW}}(f)$ of the coplanar line itself. For our setup, the function $L_{\text{CPW}}(f)$ increases approximately linearly with frequency, increasing from 0.2 dB at 1 GHz to 4 dB at 8 GHz. This leads to the linear dependence of the coefficient $l(f)$ on frequency: $l(3 \text{ GHz}) = 1.09$, $l(5 \text{ GHz}) = 0.94$ and $l(f_s) = 1$ for the reference frequency $f_s = 4$ GHz.

The measured values of the resonance frequency $f_{\text{res}}(H)$, the half-linewidth $\Delta f(H)$, and the values of the normalized magnetic susceptibility at resonance $\chi_n''(f_{\text{res}}, H)$ calculated by
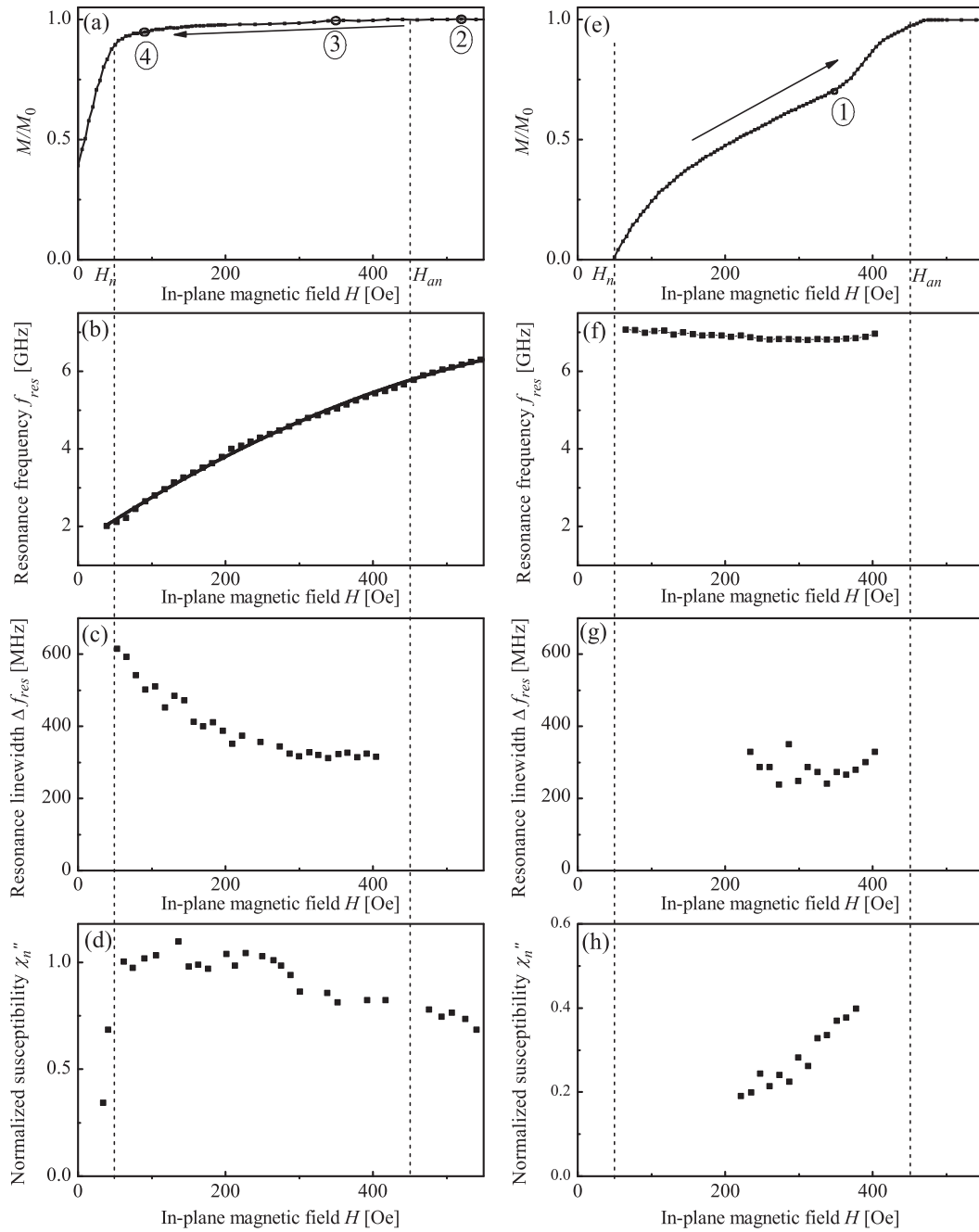
Figure 2. The measured dependences of the microwave absorption parameters (resonance absorption frequency f_{res}, half-linewidth Δf, and resonance susceptibility $\chi''_{\text{norm}}(f_{\text{res}})$) of the permalloy magnetic dot array on the in-plane bias magnetic field H. Frames (a)–(d) correspond to the quasi-uniform state of the array (the upper branch of the hysteresis loop shown by the solid line in frame (a)), while frames (e)–(h) correspond to the vortex state of the dot array (the lower branch of the hysteresis loop shown by the solid line in frame (e)). The fields of vortex nucleation $H_{\text{n}} = 50$ Oe and annihilation $H_{\text{an}} = 450$ Oe correspond to the frequencies $f_{\text{res}} = 2.1$ GHz and $f_{\text{res}} = 5.7$ GHz in the quasi-uniform ground state, respectively. The solid line in figure 2(b) is the theoretical fit obtained using Kittel's formula (5).

equation (4) using equation (2) from the measured values of $\Delta S_{21}^2(f_{\text{res}}, H)$, are presented in figure 2. Frames (a)–(d) in the left-hand column correspond to the quasi-uniform stable state of the dots (the upper lobe of the hysteresis loop), while frames (e)–(h) in the right-hand column correspond to the vortex stable state (the lower lobe of the hysteresis loop). It is clear from figure 2 that the microwave absorption properties of a dot array in the quasi-uniform and vortex stable states of the dots are qualitatively different because different spin wave modes are excited by the uniform ac magnetic field in magnetic dots existing in these two minimal energy states [4, 15].

Let us, first, discuss the microwave absorption properties of the magnetic dots existing in the quasi-uniform state. In this state, the uniform external ac field most intensively excites the lowest (main) spin mode of the dot, which has an almost uniform spatial dynamical magnetization profile with no nodes [16], and also several other modes, such as the spatially localized edge modes and the magnetostatic Damon–Eshbach-like spin wave modes.

The expressions for the resonance frequency $f_{\text{res}}(H)$, the half-linewidth $\Delta f(H)$ and the resonance susceptibility $\chi''(f_{\text{res}}, H)$ of a magnetic ellipsoid *uniformly magnetized* along the *z*-axis are well known [5, 17]:

$$\omega_{\text{res}}(H) = 2\pi f_{\text{res}}(H) = \gamma\sqrt{\left[H + 4\pi M_0\left(N_x - N_z\right)\right]\left[H + 4\pi M_0\left(N_y - N_z\right)\right]}, \quad (5)$$

$$\Delta\omega(H) = 2\pi\Delta f(H) = \alpha\gamma\left[H + 4\pi M_0\left(N_x + N_y\right)/2\right], \quad (6)$$

$$\chi''\left(\omega_{\text{res}}, H\right) = \frac{\gamma^2}{8\pi}\frac{4\pi M_0\left(H + 4\pi M_0 N_y\right)}{\omega_{\text{res}}(H)\Delta\omega(H)}. \quad (7)$$

Here M_0 is the saturation magnetization, γ is the gyromagnetic ratio, N_x, N_y, N_z are the demagnetizing factors along the symmetry axes of the ellipsoid and α is the Gilbert damping constant.

We can use expressions (5)–(7) to approximately characterize the microwave absorption in a thin ($L \ll R$, where L and R are the thickness and the radius of the dot, respectively) cylindrical ($N_x = N_z = 2b\left[\ln\left(8/b\right) - 1/2\right]$, $N_y = 1 - 2N_x$ [18], where $b = L/R$) magnetic dot magnetized in the dot plane xOz along the Oz-axis and existing in the quasi-uniform ground state (see figures 2(b)–(d)). It was shown that equation (5) yields a good accuracy for the in-plane magnetized cylindrical dots, if the dot volume averaged demagnetizing factors are used [18]. The proportionality dependence of the resonance susceptibility $\chi''(\omega_{\text{res}}, H) \propto 1/\omega_{\text{res}}(H)\Delta\omega(H)$ given by equation (7) is indeed more general than the particular saturated ellipsoid model for which it was deduced. This proportionality holds for any resonance spin eigenmode and is a result of the linear response theory.

It is clear from figures 2(b) and (c) that the dependence of the resonance frequency $f_{\text{res}}(H)$ of the lowest mode excited in the dot on the bias magnetic field H is in good qualitative agreement with the Kittel formula (5). In contrast, the experimental half-linewidth $\Delta f(H)$ actually *increases* with the decrease of the bias field H, when it should decrease according to the expression (6) for a uniformly magnetized ellipsoid. We believe that the observed increase in the linewidth is caused by the non-uniform line broadening related to the spatial inhomogeneity of the static magnetization $\mathbf{M}(\mathbf{r})$ of the dot for the bias magnetic field below the field of vortex

annihilation $H < H_{an}$. The appearance of this spatial inhomogeneity is confirmed by the micromagnetic simulations using OOMMF [19][8]. The inhomogeneous magnetization leads to the exchange-related broadening of the ferromagnetic resonance linewidth $\Delta H_{ex} \sim A/M_0\delta^2$, where δ is the scale of magnetization change [17]. Although for $H \geqslant H_{an}$ the parameter δ goes to infinity ($\delta \to \infty$), when the bias magnetic field is decreasing from H_{an} to H_n, and a spatially-nonuniform C-state is forming in the dot, the parameter δ also decreases to the values of the order of R. When the bias field is decreasing to the values of the order of the vortex nucleation field $H \approx H_n$, the parameter δ becomes comparable to the vortex core radius $\sim 20\,\text{nm}$ [15]. This should lead to a considerable broadening of the resonance absorption half-linewidth $\Delta f(H)$ as a function of the bias magnetic field H and to a corresponding decrease of the microwave susceptibility, which is inversely proportional to $\Delta f(H)$ (see equation (7)). The general linewidth broadening mechanism [17] can be translated to the language of competing energies of the stable magnetization configurations as a function of the vortex core position [15]. There is only a single energy minimum at $H \geqslant H_{an}$ corresponding to the quasi-uniform single domain (SD) state. The second (vortex) energy minimum appears at $H = H_{an}$ and becomes the lowest one at $H < H_{eq} = 290\,\text{Oe}$. The SD state is metastable at the fields $H_n < H < H_{eq}$ that contribute to magnetic relaxation, leading to the observed resonance linewidth increase at $H < H_{eq}$. Then, a new metastable C-state appears at $H = H_C = 63\,\text{Oe}$, slightly above H_n [15]. The SD state disappears at $H \approx H_n$ and there are transitions over the small energy barrier from the SD state to the C-state at $H_C > H \geqslant H_n$, which contribute considerably to the linewidth of the ferromagnetic resonance in the SD state. We would also like to note that the variation of the dot sizes as well as the edge roughness in our experimental samples was rather low (less than 5%), and did not lead to a significant extrinsic inhomogeneous linewidth broadening.

It should be noted that, although the experimental linewidth in the unsaturated magnetic dot does not obey the expression (6) for a uniformly magnetized ellipsoid, the 'ellipsoidal' expression for the resonance susceptibility (7) gives a qualitatively correct description of the experimentally observed susceptibility of the unsaturated magnetic dot in a quasi-uniform ground state (figure 2(d)) if the values of the resonance frequency $f_{res}(H)$ and half-linewidth $\Delta f(H)$ are taken from the experimental graphs figures 2(b) and (c). At small bias fields, the susceptibility decreases both due to the above-discussed exchange-related linewidth broadening and the decrease of the net magnetization of the dot in the bias field below the critical field H_n of nucleation of the magnetic vortex, while at the relatively large bias fields $H > H_{an}$, the susceptibility decreases due to the increase of the resonance frequency. The approximately constant value of the relative susceptibility in the field range 100–400 Oe (the frequency range 2.5–4.5 GHz) is, most likely, a result of inhomogeneous dot magnetization and can be explained by the combination of two factors: (i) a decrease of the dot average magnetization above the field H_n due to the formation of a strongly non-uniform C-state (the fictitious vortex core is approaching the dot border [15]) and (ii) the above-discussed exchange-related inhomogeneous linewidth broadening. The C-state can compete in energy with the quasi-uniform state only at low values of the bias field $H \leqslant 60\,\text{Oe}$ [15].

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[8] The parameters of simulations: the cell size is $5 \times 5 \times 5\,\text{nm}^3$, the gyromagnetic ratio is $\gamma/2\pi = 2.96\,\text{MHz Oe}^{-1}$, the exchange stiffness constant is $A = 1.4.10.^{-11}\,\text{J m}^{-1}$, the saturation magnetization is 800 G, and the Gilbert damping constant is $\alpha = 0.01$.

Now, let us discuss the microwave absorption properties of the magnetic dots existing in the vortex stable state in the region of the dot bi-stability ($H_n < H < H_{an}$). One of the excited modes is a low frequency vortex gyrotropic mode with an eigenfrequency that is, typically, in the sub-GHz region [4]. The analytic theory of microwave absorption exists only for the case of the excitation of the gyrotropic mode in the centered vortex state [20]. The relatively high-frequency ($f > 1$ GHz) microwave absorption in a magnetic dot existing in the *vortex* ground state is related to the excitation of spatially non-uniform dipolar spin wave modes and, in particular, to the excitation of the lowest dipolar modes with a minimal number of nodes of the microwave magnetization along the in-plane spatial (radial and azimuthal) directions. In the centered vortex state ($H = 0$), the spin wave modes can be described using the integers (n, m) indicating the number of nodes along the radial (n) and azimuthal directions [4, 16, 21]. This classification scheme can be approximately applied in small fields $H < H_n$. For instance, the azimuthal spin wave frequency doublets with indices $n = 0$, 1 and $m = \pm 1$ were observed in [12]. However, in our case, the bias field is not small (it is in the range $H_n < H < H_{an}$, i.e., it varies from 50 to 450 Oe), the vortex core is essentially off-centered and such a classification scheme is not applicable anymore. The frequencies f_{res} of the spin wave modes observed in our experiments in the off-centered vortex state depend weakly on the bias field H, and the amplitudes of these modes excited by the spatially uniform external ac magnetic field are substantially smaller than the amplitude of the main spatially uniform mode excited in the quasi-uniform ground state of the dot [13, 20] (compare also the absorption curves corresponding to the points 4 and 2 in figure 1). The experimental data presented in figures 2(f)–(h) are interpreted as excitation of the lowest dipolar spin wave mode (with no internal nodes along the radial direction) in the vortex-state dot by a uniform in-plane microwave field, which has a maximal in-plane component of the spatially averaged dynamical magnetization. Both the resonance frequency $f_{res}(H)$ and the half-linewidth $\Delta f(H)$ are practically independent of the bias magnetic field (see figures 2(f) and (g)), while the microwave susceptibility (figure 2(h)) increases with the increase of the bias field H due to the gradual increase of the net magnetization of the dot caused by the increase of H.

In summary, the resonance microwave absorption properties of circular permalloy dots were measured in the frequency range 1–8 GHz and in the in-plane bias field range 0–600 Oe, where the dots could exist in either quasi-uniform (SD or C-states) or vortex stable states. It was found that the microwave absorption properties of magnetic dots existing in the quasi-uniform and vortex states are qualitatively different. The frequency of the resonance microwave absorption in the quasi-uniform stable state increases with the increase of the bias field in agreement with the Kittel expression, equation (5), while in the vortex stable state it remains practically constant and equal to 7 GHz for the given dot parameters (see figures 2(b) and (f)). For the dots existing in quasi-uniform SD and C-stable states, a considerable linewidth broadening of up to two times, from ~300 MHz to ~600 MHz, was found when the in-plane bias magnetic field decreases from the field of a vortex annihilation H_{an} (450 Oe) to the field of a vortex nucleation H_n (50 Oe), while for the dots existing in the vortex stable state, the absorption linewidth remains practically constant in the whole interval of the bias field variation. The microwave susceptibility of the dots in the quasi-uniform state has a broad maximum in the bias field interval of 100–400 Oe and decreases at low and high values of the bias magnetic field, while in the vortex stable state, the susceptibility increases with the bias

field increase and reaches a maximum value at $H \approx H_{an}$ due to the increase in the dot average magnetization caused by the increase of H.

Due to the hysteresis in the dot magnetization stable states and substantially different microwave absorption properties in the quasi-uniform and vortex states (that can co-exist at the same value of the bias magnetic field, see e.g. absorption lines (2) and (4) corresponding to the same bias field $H = 350$ Oe in figure 1), it would be possible to use arrays of non-interacting magnetic dots in an unsaturated state for the development of dynamically reconfigurable microwave absorption materials, where the microwave properties of the materials depend on the magnetization history and could be changed dynamically through a fast remagnetization.

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