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Abstract. The goal of this contribution is to show that the hypothesis that the center of mass(CM) and the center of charge(CC) of a classical electron are two different points is only compatible with a relativistic description. The existence of two separated points is analyzed by the different dynamical behaviour of the angular momenta with respect to both points. It shows, from the classical point of view, that the angular momentum with respect to the CC of the electron satisfies the same dynamical equation as Dirac spin operator in the quantum case. In the free motion, the CC follows a helix at the speed of light, and the CM the axis of the helix. The particle, if its electromagnetic structure is reduced to a total charge e located at CC, has a magnetic moment and also an electric dipole moment with respect to the CM, like in Dirac's theory. The analysis of Dirac spin operator in the quantum case, shows that the electron is a particle where the CM and CC are necessarily different points.

1. Introduction

Real elementary particles like electrons have, among other properties, mechanical properties and also electromagnetic properties. Associated to the mechanical properties there is, from the classical point of view, a geometric point, called the center of mass $\boldsymbol{q}$, and similarly, associated to the electromagnetic properties another point $\boldsymbol{r}$, called the center of charge. Only two things can happen, that both points are exactly the same or that they are different. Although it is not explicitely assumed, the spinless point particle description considers that both points are the same. In this paper we are going to explore what physical consequences we shall obtain by assuming that perhaps the electron has some extensive, but unknown structure, such that both points are different. To our knowledge no similar analysis has been published before. What we are going to show is that, when compared with Dirac theory, the electron structure appears to be more likely associated with a classical object with two different centers than with the usual point particle assumption. The kinematical theory developed by the author leads to spinning particle models with two separate centers, as summarized in section 5.

1.1. The CM and the CC

If a real elementary particle is exactly a geometrical point then all their mechanical and electromagnetic properties would be associated to that point. If elementary particles are not exactly geometrical points, classical mechanics defines for any material system a geometrical point $\boldsymbol{q}$, called the center of mass, such that the linear momentum of the particle takes, in a nonrelativistic framework, the form $\boldsymbol{p} = m\boldsymbol{v}$, where m is the mass and $\boldsymbol{v} = d\boldsymbol{q}/dt$, or $\boldsymbol{p} = \gamma(v)m\boldsymbol{v}$, in a relativistic one, where $\gamma(v) = (1 - v^2/c^2)^{-1/2}$.

From the electromagnetic point of view, the electromagnetic structure of any charge and current distribution can always be reduced to a single point $\boldsymbol{r}$ where we locate the total charge of the system and the electric and magnetic multipoles defined with respect to that point. This point is in general arbitrary, but if the charge distribution of an elementary particle has a spherical symmetry around a point, the electric structure around that point is reduced to the value of the total charge and no other electric multipoles. If the current densities are also symmetrically distributed around that point, the magnetic multipoles also vanish. We can call this geometrical point, where the electromagnetic structure takes the simplest form of a total charge e and no multipoles, the center of charge. If we consider another point $\boldsymbol{k}$ in the particle, different from $\boldsymbol{r}$, the electromagnetic structure of the particle with respect to this point will be reduced to the same total charge e located at $\boldsymbol{k}$ and also an electric dipole $\boldsymbol{d} = e(\boldsymbol{r} - \boldsymbol{k})$ and a magnetic dipole $\boldsymbol{\mu} = e(\boldsymbol{r} - \boldsymbol{k}) \times \boldsymbol{w}/2$, provided $\boldsymbol{w}$ is the relative velocity between the point $\boldsymbol{r}$ with respect to the point $\boldsymbol{k}$.

We do not know what is the exact electromagnetic structure of the electron, how the charge and its possible internal currents are distributed. What we know is that the usual coupling of quantum electrodynamics $j^\mu A_\mu$, between the particle current field j^μ and the external potentials A_μ, is obtained in quantum field theory by the local gauge invariance hypothesis. We see that no multipole interactions with the external fields appear in this coupling. For a strict point particle of charge e, that coupling is reduced in the classical description to $-e\phi + e\boldsymbol{u} \cdot \boldsymbol{A}$, where $\boldsymbol{u}$ is the velocity of the charge, and ϕ and $\boldsymbol{A}$ the external scalar and vector potentials, respectively, defined at that point. Conversely, if we describe the classical interaction of the electron with an external field in this form, we are implicitely assuming that the interacting properties of the electron are reduced to a total charge e located at a point $\boldsymbol{r}$, where the potentials are evaluated. This would imply for an electron of unknown structure that the external fields have a smooth variation along the very small region where the charge is distributed such that the total external force can be replaced by the evaluation of the fields at the point CC where we locate the total charge, as we shall assume here.

2. Lagrangian description of a particle under an EM field

Let us consider the following Lagrangian of a particle under some external electromagnetic field

$$L = L_0 + L_I, \quad L_I = -e\phi(t, \boldsymbol{r}) + e\boldsymbol{u} \cdot \boldsymbol{A}(t, \boldsymbol{r}),$$

where e is the charge of the particle, $\phi(t, \boldsymbol{r})$ and $\boldsymbol{A}(t, \boldsymbol{r})$ are the external scalar and vector potentials, respectively, defined at the point $\boldsymbol{r}$, $\boldsymbol{u}$ the velocity of this point, and L_0 the relativistic or nonrelativistic free Lagrangian of a particle of mass m.

The Lagrangian L_0 describes the mechanical properties of the particle and L_I its interaction. L_0 is expressed as usual in terms of the CM velocity $\boldsymbol{v}$, but if there is some relationship between $\boldsymbol{q}$ and $\boldsymbol{r}$ it will be finally written in terms of $\boldsymbol{r}$ and its derivatives. The last part L_I suggests that the particle, from the electromagnetic point of view, can be considered as an object with a charge e located at point $\boldsymbol{r}$ where the external potentials are defined, and no further multipoles.

From L_0, the different mechanical properties are defined. For instance the mechanical energy H and the mechanical linear momentum $\boldsymbol{p}$, which, according to the special relativity are expressed as $H = \gamma(v)mc^2$ and $\boldsymbol{p} = \gamma(v)m\boldsymbol{v}$, respectively. The vector $\boldsymbol{v} = d\boldsymbol{q}/dt$ represents the velocity of the center of mass $\boldsymbol{q}$ and $\gamma(v) = (1 - v^2/c^2)^{-1/2}$. In the nonrelativistic limit $\boldsymbol{p} = m\boldsymbol{v}$ and $H = mv^2/2$, respectively. If the particle is exactly a geometrical point then $\boldsymbol{q} = \boldsymbol{r}$, but if it is not exactly a point these two points could be different, as we shall assume here. The interacting part suggests a spherical symmetry for its electromagnetic structure around the CC $\boldsymbol{r}$, but the mechanical part says nothing about the mass distribution. Only the existence of a point $\boldsymbol{q}$ which represents the location of the CM such that the mechanical energy and linear momentum can be expressed in terms of its velocity.

2.1. The free motion of the center of mass and center of charge

From the L_0 part we get the free dynamical equation $d\boldsymbol{p}/dt = 0$, and from the L_I part the external Lorentz force, such that the complete dynamical equation will be

$$\frac{d\boldsymbol{p}}{dt} = e(\boldsymbol{E}(t,\boldsymbol{r}) + \boldsymbol{u} \times \boldsymbol{B}(t,\boldsymbol{r})), \tag{1}$$

where $\boldsymbol{E} = -\nabla\phi - \partial\boldsymbol{A}/\partial t$, $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ and $\boldsymbol{u} = d\boldsymbol{r}/dt$. The fields are being evaluated at the CC $\boldsymbol{r}$, while the left hand side is the time derivative of $\boldsymbol{p} = \gamma(v)m\boldsymbol{v}$, or $\boldsymbol{p} = m\boldsymbol{v}$, in the nonrelativistic approach. To integrate (1) to obtain the trajectory of the CM $\boldsymbol{q}(t)$, we need to know the trajectory of the CC $\boldsymbol{r}(t)$. We have to evaluate the external fields at $\boldsymbol{r}$ and it is the velocity of point $\boldsymbol{r}$, $\boldsymbol{u}$ which is included in the magnetic force. But the CC position $\boldsymbol{r}$ will be in the neighborhood of $\boldsymbol{q}$, closely related to it, so we must make some *ansatz* about their relationship to conveniently express the linear momentum in terms of the variable $\boldsymbol{r}$ and its derivatives. Let us see first how they are related in the free case.

Let us make the analysis in an inertial reference frame, either relativistic or non-relativistic. If the particle is free $\boldsymbol{p}$ is conserved so that the CM position $\boldsymbol{q}$, moves along a straight line with a constant velocity $\boldsymbol{v}$. But, what about the trajectory of the point $\boldsymbol{r}$?

From the geometrical point of view in three-dimensional space, the trajectory of a point which follows a continuous and differentiable path, can be described as the evolution of its Frenet-Serret triad. We can parameterize the trajectory in terms of the arc length s parameter, $\boldsymbol{r}(s)$, or in terms of the time of the inertial observer $\boldsymbol{r}(t)$, if we know the kinematics of the trajectory $s(t)$. Let us consider the location of its Frenet-Serret triad at some particular time t of the inertial observer. This triad is displaced an arc length $ds = u(t)dt$ in time dt along the unit tangent vector $\boldsymbol{t}$, and rotates an angle $\kappa(t)ds$ around the binormal $\boldsymbol{b}$, and also an angle $\tau(t)ds$ around the tangent $\boldsymbol{t}$ in the same time, where $u(t)$ is the absolute value of the instantaneous velocity of the point and $\kappa(t)$ and $\tau(t)$ are the instantaneous curvature and torsion of the trajectory at time t, respectively. But if the motion is free it means that we cannot obtain a different kinematical behaviour of the CC motion at two different times. Otherwise, a different kinematical behaviour will mean that something different is happening at different times, which is contradictory with the assumption that the particle is free. The above infinitesimal displacement and angles must be independent of the time so that the CC follows a trajectory of constant curvature and torsion at a velocity of constant absolute value $u = ds/dt$, in this inertial reference frame.

The CC follows a three-dimensional helix at a constant speed. The CM seems to be the central point of the helix, i.e., the CM motion describes the axis of the helix, such that the projection of the velocity $\boldsymbol{u}$ of the CC along the axis will be the CM velocity $\boldsymbol{v}$ in this inertial reference frame.

There is a possibility that both constants κ and τ will be zero, and thus the two points $\boldsymbol{q}$ and $\boldsymbol{r}$ follow parallel straight lines with the same velocity in this frame. Different velocities will mean that both points separate from each other and this makes no sense for an elementary particle description. The CC is just displaced a constant distance from the CM and when analyzed in the reference frame where $\boldsymbol{q}$ is at rest, also $\boldsymbol{r}$ is at rest and the particle does not rotate and there is no angular momentum in this frame. Elementary matter has angular momentum and thus, what we are going to analyze is the more general situation in which the particle could possibly rotate, and therefore the constants κ and τ should be different from zero.

2.2. Nonrelativistic analysis

Let us consider the class of Galilei inertial observers, i.e., those observers related to the previous one by means of a transformation of the Galilei group. What we assume is that the above description of a CM moving along a straight line at a constant velocity and a CC describing a helix of constant curvature and torsion at a constant speed, has to be valid in every inertial reference frame, although the velocities, curvature and torsion will take different constant values in the different inertial frames. For the inertial observer of the previous analysis the curly motion of the CC, with a nonvanishing curvature and torsion, is an accelerated motion at a constant speed and therefore it is also an accelerated motion at a constant speed for the remaining inertial observers.

Since the relative velocity among inertial observers is unrestricted, let us consider some particular inertial observer which at a certain instant is at rest with respect to the CC. For this observer the velocity of the CC is $u = 0$ at this instant, and the above requirement implies that u will be zero for ever. This is impossible because the motion of the CC in this inertial frame is accelerated.

The requirement that the motion of the CC will be at a velocity of constant absolute value in every inertial frame is not verified, because this constant velocity has to be always different from zero in every frame. This means that for this requirement to be valid the velocity of the CC has to be an unreachable velocity for all inertial observers, which is not the case if the transformations among observers are those of the Galilei group.

The nonrelativistic analysis is not compatible with the existence of an elementary particle with two different CM and CC centers. For the nonrelativistic elementary particle necessarily the CM and the CC are the same point.

2.3. Relativistic analysis

Let us consider now the class of Poincaré inertial observers, i.e., those observers related to the previous one by means of a transformation of the Poincaré group.

The nonrelativistic analysis above is suggesting that the possibility of a curly motion of the CC at a constant speed in every inertial frame is allowed if the velocity of the CC is an unreachable velocity for any inertial oberver. The special theory of relativity suggests that the speed of light is a good candidate for this unreachable and constant velocity limit. In special relativity, if a point $\boldsymbol{r}$ is moving at the speed c for some particular inertial observer, then it moves with the same speed c, for the remaining ones. Only a relativistic description is compatible with the assumption of two different points CC and CM, but necessarily the CC has to be moving at the speed of light c. The free motion of a particle with two centers, implies that the CC follows a helix at the speed of light. In this case the CM velocity $\boldsymbol{v}$ will be the projection of the velocity $\boldsymbol{u}$ of the CC, along the axis of the helix, and therefore $v < c$.

2.4. Summary and discussion

This description of an elementary particle with two centers is incompatible with a nonrelativistic framework, but not in special relativity. The hypothesis of two separate points requires the existence of velocities of physical points, constant and unreachable for all inertial observers. Although the CM does not move at the speed of light, the CC does. It is accelerated and therefore classical electrodynamics implies that the particle has to radiate. From the CM point of view the particle will behave though it has a magnetic moment because there always exists a relative velocity between $\boldsymbol{r}$ and $\boldsymbol{q}$ and also an oscillating electric dipole with respect to the CM. For every observer the particle has an internal frequency ω.

The particle rotates and therefore it will have angular momentum. If the description is done in the CM frame, the motion of the CC will be a circle of constant radius R, and this implies a constant and unique angular velocity $\omega = c/R$ for all identical particles at rest, like the electrons, and also a constant and unique angular momentum with respect to the CM, which will be conserved in the free case. Otherwise, if the particle radiates, the radius will change and if the CC velocity remains constant the angular momentum of the particle must also change. For the model to be consistent, this object with a unique angular momentum has to produce no radiation whenever the CM is at rest or moving with a constant velocity. Radiation has to be produced when the particle gets some energy from the external world, i.e., when the CM is accelerated.

Classical electrodynamics is an incomplete theory because does not allow radiationless motions of finite bound systems of charged particles. We have to complement classical electrodynamics with an extra statement of no radiation for free spinning particles with a unique and unmodified angular momentum, although the CC is accelerated, provided the CM is not. Something similar to the no radiation hypothesis of the stationary orbits of quantized orbital angular momentum in Bohr's atomic formalism. We have a theory of radiation of point particles. The corresponding theory of radiation of spinning particles with two separate centers is not yet done.

We have postulated the possibility of two different centers and this leads to a unique internal relative motion between them. Are there any possibility of analysis to show if this hypothesis is right or wrong? Are there any specific observable which is able to distinguish between both points? The answer is positive. This observable is the angular momentum. Its dynamical behaviour will shed light about this conjecture.

3. The angular momentum of the particle

The angular momentum of any mechanical system is a magnitude which is defined with respect to some prescribed point. If the particle has two different characteristic points, the center of charge $\boldsymbol{r}$ and the center of mass $\boldsymbol{q}$, the angular momentum of the particle can be defined with respect to both points. Let us call $\boldsymbol{S}$ the angular momentum with respect to the CC and $\boldsymbol{S}_{CM}$ the angular momentum with respect to the CM, for some inertial observer. Even more, let us assume that $\boldsymbol{k}$ is another geometrical point of the particle, different from $\boldsymbol{q}$ and $\boldsymbol{r}$, and let us call $\boldsymbol{S}_k$ the angular momentum with respect to this point (see figure 1).

Let $\boldsymbol{J}$ be the total angular momentum of the particle with respect to the origin in this inertial reference frame. It can be written in the following alternative forms in terms of the previous angular momenta, either

$$\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{p} + \boldsymbol{S}, \quad \text{or} \quad \boldsymbol{J} = \boldsymbol{q} \times \boldsymbol{p} + \boldsymbol{S}_{CM}, \quad \text{or} \quad \boldsymbol{J} = \boldsymbol{k} \times \boldsymbol{p} + \boldsymbol{S}_k,$$

where $\boldsymbol{p}$ is the linear momentum of the particle in this inertial reference frame. The center of mass observer is defined as that inertial frame where $\boldsymbol{p} = 0$. Then, the angular momentum with respect to any point in that frame takes the same value, so that $\boldsymbol{S} = \boldsymbol{S}_{CM} = \boldsymbol{S}_k = \boldsymbol{J}$, for the center of mass observer.

If the particle is under some external electromagnetic force $\boldsymbol{F}$ evaluated and applied at the charge position $\boldsymbol{r}$, $\boldsymbol{J}$ and $\boldsymbol{p}$ are not conserved but satisfy

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{F}, \quad \frac{d\boldsymbol{J}}{dt} = \boldsymbol{r} \times \boldsymbol{F}.$$

Taking the time derivatives of the above three expressions of the total angular momentum $\boldsymbol{J}$, and taking into account these dynamical equations, we arrive at

$$\frac{d\boldsymbol{S}}{dt} = \boldsymbol{p} \times \frac{d\boldsymbol{r}}{dt}, \quad \frac{d\boldsymbol{S}_{CM}}{dt} = (\boldsymbol{r} - \boldsymbol{q}) \times \boldsymbol{F}, \quad \frac{d\boldsymbol{S}_k}{dt} = \boldsymbol{p} \times \frac{d\boldsymbol{k}}{dt} + (\boldsymbol{r} - \boldsymbol{k}) \times \boldsymbol{F},$$

because the linear momentum is pointing along the direction of the velocity of point $\boldsymbol{q}$ and not in general along the direction of the velocities of the other two points $\boldsymbol{r}$ and $\boldsymbol{k}$.

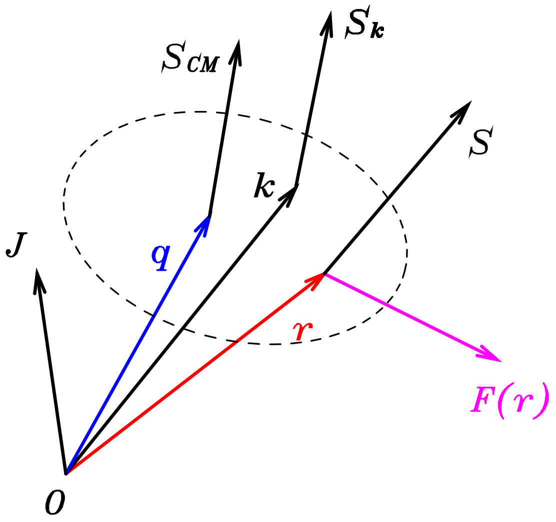

Figure 1. Different angular momenta $\boldsymbol{S}$, $\boldsymbol{S}_{CM}$, $\boldsymbol{S}_k$ and $\boldsymbol{J}$ of the electron defined with respect to different points in an inertial reference frame with origin at point O. The external force $\boldsymbol{F}$ defined at CC is also depicted. The dotted line tries to suggest some possible, but unknown, extensive structure of the electron.

These three angular momenta satisfy three different dynamical equations. In the free case $\boldsymbol{F} = 0$, and $\boldsymbol{S}_{CM}$ is conserved while $\boldsymbol{S}$ and $\boldsymbol{S}_k$ evolve in a direction orthogonal to $\boldsymbol{p}$, so that only their projections on $\boldsymbol{p}$, $\boldsymbol{S}\cdot\boldsymbol{p}$ and $\boldsymbol{S}_k\cdot\boldsymbol{p}$, respectively, are conserved.

The dynamical equation of the CC angular momentum $\boldsymbol{S}$ is independent of whether the particle is free or not. Its evolution is always orthogonal to $\boldsymbol{p}$. This means that if we compute the angular momentum of the particle with respect to some point, the dynamical behavior of this angular momentum will also give us information about whether this point is the center of charge, the center of mass or a point different from them.

If the particle is not free but both points are the same $\boldsymbol{r} = \boldsymbol{q}$, and also $\boldsymbol{S} = \boldsymbol{S}_{CM}$ and thus these angular momenta (but not $\boldsymbol{S}_k$) are necessarily conserved. But, conversely, if the angular momentum with respect to the CC or with respect to the CM is not conserved it means that this point $\boldsymbol{r}$ is necessarily a different point than the center of mass $\boldsymbol{q}$.

The CC angular momentum $\boldsymbol{S}$ does not satisfy a torque dynamical equation like $\boldsymbol{S}_{CM}$. It is not conserved for a free particle. It seems to precess around the direction of the linear momentum and its dynamical equation is independent of the external force.

The analysis of a particle with two centers produces a unique description of a motion of the CC at the speed of light. The angular momentum with respect to the CC satisfies the same dynamical equation than Dirac's spin operator in the quantum case. This is going to be compared with Dirac's analysis of the electron.

4. Dirac Analysis

In his original papers in 1928, Dirac [1],[2] analyzes an electron interacting with an external electromagnetic field, through a minimal coupling and arrives to the Hamiltonian

$$H = (\boldsymbol{p} - e\boldsymbol{A}(t,\boldsymbol{r}))\cdot c\boldsymbol{\alpha} + \beta mc^2 + e\phi(t,\boldsymbol{r}),$$

where $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$ and $\beta = \gamma^0$, are Dirac's hermitian matrices. In the original papers Dirac uses a different notation for the above matrices but we have kept today's more accepted one. The usual minimal coupling interaction, where the potentials, ϕ and $\boldsymbol{A}$, are defined at point $\boldsymbol{r}$, suggests that point $\boldsymbol{r}$ represents the location of the CC. But, is it also the location of the CM?

When computing the velocity of point $\boldsymbol{r}$, Dirac obtains

$$\boldsymbol{u} = \frac{d\boldsymbol{r}}{dt} = \frac{i}{\hbar}[H, \boldsymbol{r}] = c\boldsymbol{\alpha},$$

irrespective of whether the particle is free or not. The eigenvalues of the hermitian matrices $\boldsymbol{\alpha}$ are ± 1. It writes on page 262 of his book [3]: "*... a measurement of a component of the velocity of a free electron is certain to lead to the result $\pm c$. This conclusion is easily seen to hold when there is a field present*".

This point $\boldsymbol{r}$ is moving at the speed of light. In Dirac's Nobel lecture [4] he says: "*It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. ... one must believe in this consequence of the theory, since other consequences of the theory ..., are confirmed by experiment.*"

To see if point $\boldsymbol{r}$ represents also the location of the CM, let us analyze the dynamical behaviour of the angular momentum of the electron with respect to this point. If it represents the location of the CM, it will be conserved in the free or interacting motion. Otherwise, the CM will be a different point than the point $\boldsymbol{r}$. The total angular momentum of the electron with respect to the origin of the observer frame is

$$\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{p} + \boldsymbol{S}, \quad \boldsymbol{S} = \frac{\hbar}{2}\begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix},$$

where Dirac spin operator $\boldsymbol{S}$, written in terms of Pauli matrices, represents the angular momentum of the electron with respect to the point $\boldsymbol{r}$. Both parts $\boldsymbol{r} \times \boldsymbol{p}$ and $\boldsymbol{S}$ are not separately conserved for the free electron. In the introduction of [1], Dirac writes: "*The most important failure of the model seems to be that the magnitude of the resultant orbital angular momentum of an electron moving in an orbit in a central field of force is not a constant, as the model leads one to expect.*"

The spin part $\boldsymbol{S}$ satisfies

$$\frac{d\boldsymbol{S}}{dt} = \frac{i}{\hbar}[H, \boldsymbol{S}] = \boldsymbol{p} \times c\boldsymbol{\alpha} = \boldsymbol{p} \times \boldsymbol{u}$$

even under the external interaction. This no torque dynamical equation for an angular momentum suggests that the point $\boldsymbol{r}$ where the external fields are defined is a different point than the CM of the electron.

The linear momentum is not along this velocity $\boldsymbol{u}$ but is related to some average value: "*... the x_1 component of the velocity $c\alpha_1$, consists of two parts, a constant part $c^2p_1H^{-1}$, connected with the momentum by the classical relativistic formula, and an oscillatory part, whose frequency is at least $2mc^2/h$,...*". Point $\boldsymbol{r}$ is not the position of the CM. This frequency predicted by Dirac for the motion of the point $\boldsymbol{r}$ is just twice de Broglie's postulated frequency.

When expanding the Hamiltonian in powers of $\boldsymbol{p}$, i.e., which could be intrepreted as the expression of the energy in terms of the CM motion, he finds, in addition to the kinetic energy terms and the interacting term $e\phi$, two new interaction terms:

$$\frac{e\hbar}{2mc}\boldsymbol{\Sigma} \cdot \boldsymbol{B} + \frac{ie\hbar}{2mc}\boldsymbol{\alpha} \cdot \boldsymbol{E} = \boldsymbol{\mu} \cdot \boldsymbol{B} + \boldsymbol{d} \cdot \boldsymbol{E}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}$$

He says: "*The electron will therefore behave as though it has a magnetic moment $(e\hbar/2mc)\boldsymbol{\Sigma}$, and an electric moment $(ie\hbar/2mc)\boldsymbol{\alpha}$. The magnetic moment is the just assumed in the spinning electron model. The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether the electric moment has any physical meaning ...*".

In his book [3] gives the same expression but he never mentioned, even in subsequent works like in the Nobel conference [4], the existence of this electric dipole. He analyzes the magnetic interaction but devotes no single line to the electric dipole interaction, which has appeared on the same footing as the magnetic one. He disliked that the electron would have an electric dipole structure. The absolute value of this term is the charge e times a distance $\hbar/2mc$. The operator $(i\hbar/2mc)\boldsymbol{\alpha}$, represents the relative position quantum operator of the CC with respect to the CM, as has been shown in [5]. It simply means that the magnetic dipole and the electric dipole represent the electromagnetic electron structure with respect to the CM, provided its structure with respect to the CC is that of a total charge e, without further multipoles.

The electron has two different centers separated by half Compton's wavelength. The motion of the CC around the CM, in the center of mass frame, is at a frequency twice the frequency postulated by de Broglie and at the speed of light. In the quantum case, the free electron does not radiate.

5. Kinematical theory

In the kinematical formalism for describing spinning particles developed by the author [5], this assumption about the existence of two separated centers is not postulated. What is postulated is that an elementary particle cannot have excited states and its internal structure cannot be modified by any interaction. The mathematical requirement of this postulate is that the boundary variables of its classical Lagrangian description span necessarily a homogeneous space of the Poincaré group. The description of a classical elementary spinning particle in this formalism, is in terms of a single point $\boldsymbol{r}$ where the external potentials are defined. What is found is that the Lagrangian which describes an elementary spinning particle, in the most general case, has to be expressed in terms of the velocity and acceleration of this point, and thus dynamical equations are fourth order differential equations for the variable $\boldsymbol{r}$. The linear momentum is found to be not along the velocity of this point. The angular momentum with respect to this point $\boldsymbol{S}$, satisfies the same dynamical equation as Dirac's spin operator. General spinning particles have a CM which is different point than the CC although for its dynamical description it is sufficient to give the evolution of a single point $\boldsymbol{r}$, the CC.

The only spinning model in this kinematical formalism which satisfies Dirac's equation when quantized corresponds to the model such that the velocity of point $\boldsymbol{r}$ is the speed of light [6]. Quantization of all other spinning models does not produce Dirac's equation. The classical description of the electron at rest is given in figure 2. It is a mechanical system of six degrees of freedom. Three $\boldsymbol{r}$, represent the location of the CC, and another three $\boldsymbol{\alpha}$, represent the orientation of a local cartesian frame located at the CC position and which is not depicted in the figure. What is depicted is the angular velocity $\boldsymbol{\omega}$ of this local frame. The radius of this motion is $R = |S_z|/mc = \hbar/2mc$, half Compton's wavelength, and the angular velocity in this frame $\omega = 2mc^2/\hbar$ is twice De Broglie's postulated frequency. In this frame, the total angular momentum $\boldsymbol{S} = \boldsymbol{S}_{CM}$ is conserved and the dynamical equation satisfied by the CC of the particle is

$$\boldsymbol{r} = \frac{1}{mc^2}\boldsymbol{S}\times\boldsymbol{u}, \quad \boldsymbol{u} = d\boldsymbol{r}/dt, \quad u = c.$$

For the antiparticle the motion of the CC is the reversed one, with the same orientation of the spin. The total angular momentum is the addition of two terms, $\boldsymbol{S} = \boldsymbol{Z} + \boldsymbol{W}$, one $\boldsymbol{Z}$ related to the relative motion of the CC around the CM, known as the *Zitterbewegung*, and another $\boldsymbol{W}$ related to the change of orientation and thus along the angular velocity $\boldsymbol{\omega}$.

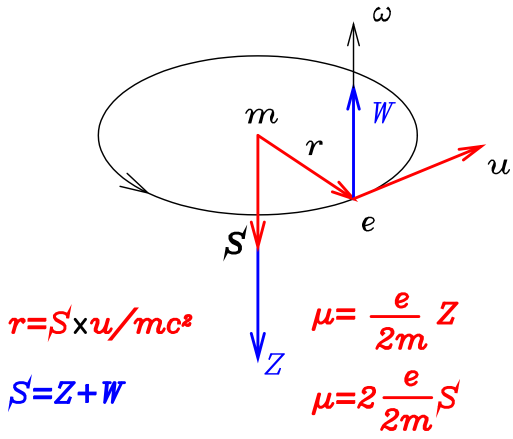

Figure 2.

In this formalism, the separation between the CM and CC is responsible for a classical interpretation of the formation of bound pairs of spinning electrons [7] (see figure 3) and the clarification of the mechanism of tunneling [8]. Quarks are considered Dirac particles. Since Dirac spin operator represents the angular momentum of the quark with respect to the CC, it is impossible that the addition of the three Dirac spin operators of the three quarks gives rise to the spin of the proton. This apparent failure, (see figure 4) known in the literature as the proton spin crisis, has been recently pointed out [9].

6. Conclusions

In this paper we have shown that the hypothesis of two separate centers for elementary spinning particles is consistent only with special relativity and that these objects have a unique angular momentum that cannot be modified. In the free motion, the CC moves along a helix at the speed of light and the CM describes the axis of the helix. The quantization of this model satisfies Dirac's equation, such that Dirac spin operator represents the angular momentum with respect to the CC.

Acknowledgments

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References

[1] Dirac PAM 1928 The Quantum Theory of the Electron, *Proc. Roy. Soc. Lon.* **A117**, 610
[2] Dirac 1928 The Quantum Theory of the Electron. Part II, *Proc. Roy. Soc. Lon.*, **A118**, 351
[3] Dirac PAM 1967 *The Principles of Quantum mechanics* (Oxford: Oxford Univ. Press.) 4th ed
[4] Dirac PAM 1933 Nobel Lecture: Theory of Electrons and Positrons, `http://www.nobelprize.org/nobel_prizes/physics/laureates/1933/dirac-lecture.html`
[5] Rivas M 2001 *Kinematical theory of spinning particles* (Dordrecht: Kluwer)
[6] Rivas M 1994 Quantization of generalized spinning particles. New derivation of Dirac's equation, *J. Math. Phys.*, **35**, 3380

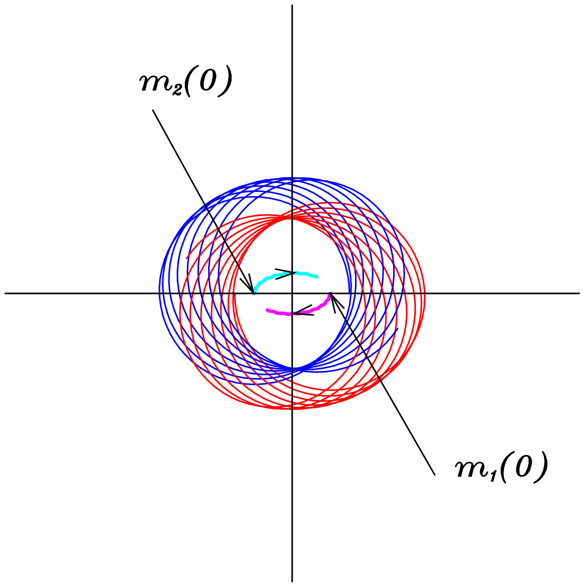

Figure 3. Bound motion of the CC's (red and blue) and CM's (light red and blue) of two spinning electrons with parallel spinsut. This bosonic system is stable under external electric fields but it is destroyed with a transversal magnetic field.

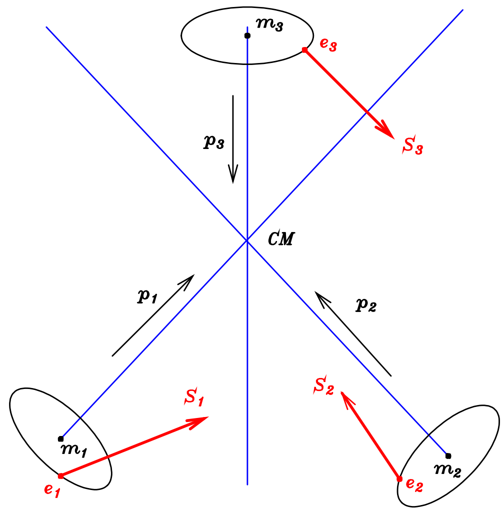

Figure 4. Proton at rest showing its CM and the coplanar motion of the CM's of the three quarks where the $\boldsymbol{S}_i$ (in red) represent the angular momenta of each quark with respecto to the corresponding CC, $\boldsymbol{r}_i$. Since the $\boldsymbol{S}_i$ are represented by the Dirac spin operators in the quantum case, it is impossible that the addition of the three Dirac spin operators $\boldsymbol{S}_i$ gives rise to the total angular momentum of the quarks with respect to the CM of the proton. The addition of the three terms $\boldsymbol{r}_i \times \boldsymbol{p}_i$ is left

[7] Rivas M 2003 The dynamical equation of the spinning electron, *J. Phys. A: Math. and General*, **36**, 4703. (arXiv:physics/0112005).
[8] Rivas M 1998 Is there a classical spin contribution to the tunnel effect?, *Phys. Lett. A*, **248**, 279
[9] Rivas M 2011 A lacking term in the proton spin, *J. Phys. Conf. Series*, **295**, 012040