LEIBNIZ ON PRIVATIVE AND PRIMITIVE TERMS

Wolfgang LENZEN*

ABSTRACT

We first present an edition of the manuscript LH VII, B 2, 39 in which Leibniz develops a new formalism in order to give rigorous definitions of positive, of privative, and of primitive terms.

This formalism involves a symbolic treatment of conceptual quantification which differs quite considerably from Leibniz's "standard" theory of "indefinite concepts" as developed, e.g., in the "General Inquiries". In the subsequent commentary we give an interpretation and a critical evaluation of Leibniz's symbolic apparatus. It turns out that the definition of privative terms and primitive terms lead to certain inconsistencies which, however, can be avoided by slight modifications.

<u>THE TEXT</u> (LH IV, 7 B 2, 39)*

l terminus ut A   ł oppositum termini seu non-A

b terminus positivus   ƀ terminus privativus

bƀ terminus partim positivus partim privativus

Videndum an in pronuntiando liceat opposita exprimere per aspirationes. Terminus positivus est qui dicit perfectionem, privativus qui limitationem. Sed fortasse pro termino positivo et privativo exprimendo, non erit opus novo signo. Est enim positivus, in quo sufficienter resoluto non reperitur ł seu negativum. Privativus in quo sufficienter resoluto non reperitur positivum. Mixtus in quo reperitur utrumque.

⊓ terminus qui continet aliquem terminum talem, qui sequetur vel jam affuit.

l l terminus ex duobus compositus qui sequentur vel affuere; sed videndum quomodo exprimatur esse diversos. Fortasse diversi ad diversos relati distingui possunt vocalibus. Videndum quomodo exprimatur terminum aliquem esse nullo modo compositum, vel non esse compositum, sed omnino primitivum.

Forte poterit ⊓ significare terminum qui aliquem alium, quemcunque continet, seu l l compositum ex duobus terminis; quando non adjicintur vocales, nisi forte i vel si mavis scheva. Sed quando vocales aliae adjicintur intelligetur continens talem terminum.

⊓ A continens B l l AB

ł⊓ erit terminus negans continens asserentem seu non A continens B.

ł—ł non-(terminus continens terminum)** seu non (A continens B)

⊓̸ terminus non continens terminum seu A non continens B

⊓ł terminus continens oppositum alicuius termini. A continens non B.

l̇ ł̇ vel l̈ ł̈ A non A, B non B.

Ad regulas scriptionis pertinet ut ł̇ idem sit quod l̇ et l̇ l̇ idem quod l̇.

Ut ⊓̸ł est l non continens ł et ⊓̸ l non continens l; ita l✱l poterit esse l excludens l, sec hoc idem est quod ⊓ł, seu l continens ł seu A continens non B.

Perfectius tamen erit hoc totum, si potius exclusionis signum ex contenti oppositi signo fiat. Imo ipsum exclusionis signum est ⊓ł ut proinde altero non sit opus, verbi gratia l̇ l̇⊓ł̈ ł̈ significabit: A excludens B seu continens non B; ubi tamen l̇ et ł̈ adjici erit non necessarium.

Primitivus erit A non continens Y positiva, quod sic scribere licebit: ḃ⊓̸b̈ ita ut Ր significet Y, et б Y positivum. Itaque ḃ⊓̸b̈ est primitivus sed intelligi debet diversus a b, nempe terminus est primitivus qui nullum continet terminum

praeter seipsum; seu qui terminum positivum alium a seipso non continet, ubi tamen aliquod adhuc indicari vel intelligi opus est, nempe ḃ et b̈ non posse sumi pro eodem. Quemadmodum et vicissim aliquando indicari debet ḃ et b̈ licet pro diversis indicata esse eadem. Hoc fortasse sic indicabitur ḃ ∞ b̈ et ḃ ∞
(*bricht ab*)

Forte satius erit sic precedere: lCl significabit l continens l seu terminus continens terminum. Quod si malimus continere per coincidentiam explicare ita ut l̇ ∞ r̈ l̇ sit terminum l̇ coincidere termino l̈ cum termino aliquo (r̈). Quod si linea superducatur, ea significabit: est, et cum obelo non est: l̇ l̈ significat l̇ l̈ est, et l̇ l̈ significat non est: ita ut linea ex termino faciat propositionem; l̇ ≁ r̈ r̈ si l̇ ≁ l̈ l̇ es primitivus

Utile erit l̇ scribere per L[1] quia ipse est subjectum et fiet primitivus L̇ ≁ r̈ r̈ si L̇ ≁ l̈, seu primitivus: (L̇ ≁ r̈) ∞ l̈ (L̇ ≁ r̈ r̈)••• sed separatim adhuc exprimendum omnes terminos esse positivos.

Sit A non XY posito A non X et posito X et Y positivis, erit A primitivus.

Optimum erit definitiones persequi per literas, deinde non difficile erit aptos excogitare characteres.

Terminus A, B. Terminus indefinitus Y non A; non Y; terminus ipsi A vel Y contradictorius, seu si A ∞ non B erunt A et (B) contradictorii et non B dicitur negans, B affirmans.

Terminus positivus videtur esse qui quatenus continet non A, eatenus continet non non B; seu cuius quodlibet non destruitur per aliud non.

∞ , non, et similes notae etiam possunt haberi pro terminis; itaque ∞ significat idem quod Y. Sic non est non Ens, item non verum.

Terminus falsus est qui continet Y non Y. Verus qui non est falsus.

Wolfgang LENZEN

TEXT-CRITICAL APPARATUS

Line 4: liceat: (1) privatio *(bricht ab)* (2) opposita

Line 6: fortasse: (1) non opus erit (2)pro ...

Line 13: /qui sequentur vel affuere/ *erg.L.*

Line 14: diversos: (1) et quomodo (2) Fortasse ...

Line 1 : relati: (1) exprim *(bricht ab)* (2) distingui ...

Line 16: /nullo modo/ *erg.L.*

Line 26: (1) ┌/┐ (2) ┌─┐ (3) ┼┼ ...; /seu ... B/ *am Rande erg.L.*

Line 27: continens: (1) non B (2) B *verb. Hrg.*

Line 28: A non continens: (1) non B (2) B *verb. Hrg.*

Line 29: continens: (1) non terminum (2) oppositum ...

Line 31: l̇ ł̇ : (1) terminus continens oppositum A conti *(bricht ab)*;
(2) vel *(streicht Hrg.)* (a) l (b) l̈ l (c) vel ...

Line 35: ita: (1) l* (2) l* eri *(bricht ab)* (3) l*l poterit ...

Line 36/37: B.: (1) Perfectius tamen erit hoc *(bricht ab)* ; (2) Perfecti
(3) Perfectius ...

Line 37: potius: (1) excludentis (2) exclusionis ...

LIne 38: fiat.: (1) Forte ipsum (2) Imo ipsum ...

Line 39: ┌─ł: (1) Non male (2) ut ...

Line 40: excludens B: (1) <—> (2) seu ...

Line 42: erit: (1) non continens (2) A non ...

Line 43: licebit: (1) l (2) A l (3) l (4) ɓ̃ (5) ɓ─ɓ (6) ɓ/ɓ̃ nempe
(a) ɓ̃ (b) ɓ (c) ɓ̃ est terminus indefinitus. (7) ɓ/ɓ̃
(8) /ɓ/ɓ̃/ *verb. Hrg.* ...

Line 44: itaque: (1) ɓ̃ (2) ɓ/ɓ̃ (3) /ɓ/ɓ̃/ *verb. Hrg.* est primitivus (a) ɼ (b) sed ...debet: (a) diversum (b) diversus ...

Line 45: nempe: (1) l (2) ɼ est terminus (3) terminus ...

Line 47: tamen: (1) adhuc (2) aliquod ... indicari /vel inteligi/ *erg.L.*

Line 50: indicabitur: (1) ɓ∞ (2) ɓ∞ (3) ɓ∞ɓ ...

Line 52: procedere: (1) ɓCl (2) lCl significabit (a) ɓ (b) l (ba) continere (bb) continens l ...

Line 53/53: continere: (1) ex *(bricht ab)* (2) per ...

Line 54: l̇∞ɼ̈ l̈ : (1) fit (2) sit ...

Line 55: aliquo: (1) ɼ (2) ɼ̈ *verb. Hrg.*

Line 58: propositionem; /sed/ *streicht L.*

Line 59: l∞̸ r r : (1) et (2) si l∞̸ l (a) vel si l (b) l ...

Line 59/60-63: primitivus: (1) seu (2) /Utile ... (a) L (b) L *verb. Hrg.* ...positivos/ *am Rande erg. L.*

Line 62-63: ∞ l (L ∞̸ r r): (1) Pro L substi *(bricht ab)* (2) sed ... exprimendum: (a) L es *(bricht ab)* (b) omnes ...

Line 64: Sit A non ∞: (1) Y (2) XY posito (a) l non (b) A non ∞ (ba) Y (bb) X (bba) erit A prim *(bricht ab)* (bbb) et posito ...

Line 69: vel Y: (1) oppositus (2) contradictorius ...

Line 70: A et (1) Y (2) B *verb. Hrg.*; ... negans, (a) A (2) B affirmans...

Line 71/2: affirmans: (1) Si A   XY et *(bricht ab)* (2) A non ∞ XY posito X non ∞ A erit A primitivus. (3) A (4) Terminus ...

Line 72-73: qui: (1) si (2) quatenus ... non A, (a) continet non non A quod (b) eatenus continet ...

Line 76: ∞ : (1) idem est quod idem et *(bricht ab)* (2) significat idem quod (a) <—> (b) Y Sic non est non Ens (a) seu et *(bricht ab)* (b) item ...

COMMENTARY

This fragment is remarkable because of two points: (1) Leibniz develops an (although incomplete) formal system of concept logic whose symbolic operators largely differ from his other drafts of a **universal calculus**[1]; (2) Leibniz tries to give strictly formalized definitions of privative and primitive concepts. At the beginning of the essay Leibniz introduces:

- the symbol l as a variable for arbitrary concepts or **terms**;
- b as a variable for **positive** concepts; and
- the symbol ' - ' as the operator of term-**negation**.

Accordingly a **privative** term can simply be expressed as the negation, b̶ , of a positive term.[2] Leibniz is wondering whether one might define positive and privative terms also without the help of the symbol b ; such a definition should be based on the consideration that a term l is positive if its analysis will not bring to light any negated term contained in l. This problem will be taken up towards the end of the fragment. Next Leibniz introduces:

- the raised bar between two terms as a symbol for the relation of conceptual **containment**: $\overline{l\quad l}$; and
- juxtaposition to symbolize the **conjunction** of two terms.

This formal representation of terms is somewhat ambiguous since several occurrences of the same symbol 'l' do not necessarily denote the same concept. In particular 'l l' may be taken to represent either 'AA' or 'AB', and the different expressions 'A containing B' and 'B containing A' would both be formalised as $\overline{l\quad l}$. Therefore Leibniz sets himself the task of finding a way for indicating the distinctness of terms, e.g. by means of "vowels". And he also notes (lines 15-17) the task of determining when a term is primitive, i.e. not constituted of other terms; this will further be investigated from lines 42 onwards.

The subsequent passage (lines 18-22) is somewhat obscure. The handwriting does not crearly reveal whether the signs immediately after the word 'seu' mean 'l l' or whether they are merely the result of deleting some other letters. In the latter case Leibniz would be considering using one and the same schema $\overline{l\quad l}$ to express either "a term which contains some other term" or (seu) a "term composed out of two terms". The subsequent qualification "if (no) vowels are added" might then be interpreted as the suggestion to **distinguish** both senses by means of "vowels". Anyway in line 23 Leibniz returns to the earlier symbolism which has '$\overline{l_1\quad l_2}$' for 'l_1 containing l_2' and '$l_1 l_2$' for the conjunction of both terms. This is more satisfactory in view of the subsequent theory of negation which requires to interpret $l_1\quad l_2$ as a **proposition** while $l_1 l_2$ itself crearly as a **term.**

In lines 24-30 Leibniz deals with the different ways of negating the relation of conceptual containment. Neglecting the trivial cases of double negation, there are $2^3 - 1$ different ways of inserting negation operators into the schema $\overline{l\quad l}$ to express that a positive or a negative term contains or does not contain another positive or negative term. Leibniz begins with the case where a negative term contains a positive one, $\overline{\not{l}\quad l}$: "non A containing B"; the reverse case, $\overline{l\quad \not{l}}$, where a positive term contains a negative one: "A containing non B", is mentioned in lines 29/30 and will further be investigated in lines 34-41. The corresponding containment between two negative terms, $\not{l}\!\sqcap\!\not{l}$, is omitted by Leibniz.

Also the different forms of negating the relation of containment among (positive or negative) terms are not explored very systematically by Leibniz. He first attempts to formalise "non-(term containing term)" rather ambiguously as $\not{l}\!-\!\not{l}$. On the one hand, this formula, or more precisely $\not{l}_1\!-\!\not{l}_2$, might be taken to express a **term**. In this case it could denote either the conjunction of the two negative terms $\not{l}_1$ and $\not{l}_2$, or the negation of the conjunction $l_1 l_2$. On the other hand, $\not{l}_1\!-\!\not{l}_2$ may be interpreted as a proposition saying that l_1 does not contain l_2. This would have to be paraphrased as "non-(A containing B)" but not, as Leibniz erroneously puts it in the margin, as "non-(A containing non [!] B)". In the subsequent sentence Leibniz formalises "term not containing term" in the less ambiguous (and more "natural") way $l\!\not\sqcap\!l$. However, in the margin he once again gives the incorrect paraphrase "A not containing non [!] B", and he also forgets to formalise the remaining cases corresponding to "A/non-A non continens B/non-B.

As from line 31 onwards, Leibniz adds dots ("scheva") to the symbol l in order to distinguish different terms l_1, l_2, l_3, etc. He then formally represents the contradictory concepts 'A non A' and 'B non B' as '$l_1\not{l}_1$' and '$l_2\not{l}_2$', respectively. Next, he states the simple laws of double negation:

(1) $\not{\not{l}}_1 \infty\; l_1$

and of idempotence of conjunction:

(2) $l_1 l_1 \infty\; l_1$.

Then Leibniz returns to the formal representation of the (universal negative) proposition 'l_1 excludes l_2'. He soon recognizes that in accordance with the syllogistic principle of obversion 'l_1 excludes l_2' is tantamount to 'l_1 contains non-l_2' so that it is not necessary to introduce a new symbol '$\divideontimes$'. It remains unclear why in line 40 Leibniz first writes '$l_1 l_1\!\sqcap\!\not{l}_2\not{l}_2$' instead of '$l_1\!\sqcap\!\not{l}_2$', nothing himself one line later that (the left occurrence of) 'l_1' and (the right occurrence of) '$\not{l}_2$' are redundant.

In the subsequent passage Leibniz adresses again the main task of defining the primitiveness of terms. For this sake he introduces a new symbol $\sim$ which appended to the term variable l yields the **indefinite term** $\tilde{l}$. As is known from the GI and from several fragments of C, Leibniz uses such indefinite terms (usually denoted by X, Y, Z, ...) as a kind of term-quantifier, primarily functioning as an **existential** quantifier and only seldom as a universal quantifier.[3] The text-critical apparatus reveals that Leibniz makes a series of efforts to formally define the primitiveness of terms before he ends up with the expression $\overline{l_1 \not\; \tilde{l}_2}$. This formula expresses that the positive term l_1 does not contain (any) term of the type $\tilde{l}_2$, i.e. any **privative** term $\not\tilde{l}_2$. However, immediately afterwards Leibniz explains that a (positive) term l_1 is primitive if and only if it does not contain any **positive** term l_2 besides itself ("terminum positivum alium a seipso"). Disregarding for a moment the requirement that l_2 must be different from l_1 , Leibniz's definition of primitivity thus has to be corrected at least in the following way:

(3) l_1 is primitive if and only if (for short, iff): (for every $\tilde{l}_2$ different from l_1) $\overline{l_1 \not\; \tilde{l}_2}$.

In lines 48-50, Leibniz looks for - but apparently fails to find - a satisfactory expression for the distinctness or nondistinctness of terms l_1 , l_2. Then he suddenly changes the topic and attempts the new symbolization $l_1 \mathrm{C} l_2$ for the relation of conceptual containment. Immediately afterwards, however, he dispenses with this relation in favor of conceptual identity (or coincidence) ∞ . The corresponding law says that l_1 contains l_2 iff l_1 coincides with l_2 plus some other term $\tilde{l}_3$ ("termino l_2 cum termino aliquo $\tilde{l}_3$ "), i.e.:

(4)[4] $l_1 \mathrm{C} l_2$ iff there is some (indefinite) concept $\tilde{l}_3$ such that $l_1 \infty l_2 \tilde{l}_3$.

In the next sentence Leibniz introduces another element into his symbolic system of term logic, **viz.** the operator 'est'. At first sight the expression obtained by drawing a line above a conjunctive term $l_1 l_2$ appears to be the same as the symbolic representation of 'l_1 contains l_2 ' gives in the first part of the essay. But a closer inspection reveals the following difference. Whereas the line in '$\overline{l_1 \; l_2}$ ' usually is connected with the top of the left term l_1 so as to form kind of a "roof" for the right term l_2 (being contained in the former), the line in

'$\overline{l_1 \quad l_2}$' symmetrically embraces or encloses both terms. Moreover, the proposition 'AB est' is interpreted by Leibniz in several other logical essays always as 'AB is possible' ("est possibile", "est Ens", "est res") and thus distinguished from 'A est B', i.e. 'A contains B'. The logical relation between the operator of conceptual containment on the one hand and the conceptual possibility on the other was formulated e.g. in § 200, GI, as follows: *"Si dicam AB non est, idem est ac si dicam A continet non-B"*. In the symbolic language of the present study, this law takes the form:

(5) $\overline{l_1 \quad l_2}$ iff $\lceil l_1 \quad \bar{l}_2 \rceil$

However, in the remainder of the essay the possibility or self-consistency of concepts plays no role at all.

Being equipped with the relation of conceptual identity, ∞ , Leibniz is now able to formulate the condition of the distinctness of b_1 and b_2 (as required in the definition of primitiveness) simply by $l_1 \not\infty l_2$. Thus "l_1 is primitive" is reformulated in line 59 as "$l_1 \not\infty \tilde{l}_2 \; \tilde{l}_3$ if $l_1 \not\infty l_2$" which contains a minor slip, however. In view of (4), $l_1 \not\infty \tilde{l}_2 \; \tilde{l}_3$ is tantamount to $l_1 \not\subset \tilde{l}_2$, i.e. to $\lceil l_1 \quad \tilde{l}_2 \rceil$. Therefore the quoted formula is meant to express that a primitive term l_1 does not contain **any** term l_2 besides l_1 itself. Accordingly l_2 must be taken as an **indefinite** term and hence be symbolized as $\tilde{l}_2$. This, incidentally, is also evident from Leibniz's subsequent paraphrase "A not ∞ XY provided that A not ∞ X". In sum, then, Leibniz's second definition of primitivity given in line 59 amounts to:

(6) l_1 is primitive iff (for every $\tilde{l}_2$): if $l_1 \not\infty \tilde{l}_2$, then $l_1 \not\infty \tilde{l}_2 \; \tilde{l}_3$.

In the subsequent passage Leibniz presents an even more formalised condition by requiring: "$L_1 \not\infty \tilde{l}_2 \infty l_3 (L_1 \not\infty \tilde{l}_2 \; \tilde{l}_3)$". This formula is quite puzzling. The fact that the main term, l_1 , is now expressed by a capital 'L_1' "because is the subject" is of no great importance. The interesting point rather is the attempt to condensate proposition (6) which has the structure 'if α then β' into something like the equation '$\alpha \infty \beta$'.

In GI and in some later fragments, Leibniz stressed the possibility of conceiving **propositions** about concepts ("incomplex terms") themselves as **"complex terms"**. In particular the implication between propositions may be regarded as structurally equivalent to the containment among terms so that 'if α then β' can be represented as 'α contains β'[5]. Accordingly the biconditional 'α if and only if β', which corresponds to the mutual containment of both terms, may be formalised as 'α ∞ β'. Now, for logical reasons the proposition β in the definiens of (6), i.e. $l_1 \not\infty \tilde{l}_2 \tilde{l}_3$, entails the proposition α, i.e. $l_1 \not\infty \tilde{l}_2$, because whenever l_1 is different from $\tilde{l}_2$ $\tilde{l}_3$, for arbitrary $\tilde{l}_3$, then l_1 is in particular different from $\tilde{l}_2$ $\tilde{l}_2$, i.e. from $\tilde{l}_2$ itself! Hence the **implication** in the definiens of (6) may well be strenghtened into an **equivalence** and thus be formalized as:

(7) L_1 is primitive iff (for every $\tilde{l}_2$ and every $\tilde{l}_3$):
$((L_1 \not\infty \tilde{l}_2) \infty (L_1 \not\infty \tilde{l}_2\ \tilde{l}_3))$.

It is likely to assume that the left occurrence of 'l_3' in the subformula 'l_3 $(L_1 \not\infty \tilde{l}_2\ \tilde{l}_3)$' in line 62 of the manuscript is either a slip of the pen or was mistakenly not deleted from an earlier version of the formula. Anyway (7) correctly represents Leibniz's ideas about the primitivity of L , provided we add the requirement (lines 62/63) that "all the terms are positive":

(8) L_1 is primitive iff L_1 is positive and (for every positive $\tilde{l}_2$, $\tilde{l}_3$):
$((L_1 \not\infty \tilde{l}_2) \infty (L_1 \not\infty \tilde{l}_2\ \tilde{l}_3))$.

This may be simplified by requiring that a primitive term, L_1, is never conjunctively composed of other, positive terms l_2 l_3, except for the trivial composition $L_1 = L_1 L_1$. Hence - as was already formulated in (3) - a primitive term does not contain any "positive term different from itself".

In order to obtain a really satisfactory definition of **primitive terms,** then, either (3) or (8) has to be supplemented by an appropiate definition of **positive terms.** Unfortunately, Leibniz's concluding attempt to define positiveness is not without problems. Taken literally, the statement (lines 72-73) that a term C "seems to be positive" iff "insofar as it contains non-A, it contains non-non-B" would have to be paraphrased as follows:

(9) l_1 is positive iff for every l_2: if l_1 contains $\not{l}_2$, then there is some l_3 such that $l_2 \infty \not{l}_3$.

However, (9) is trivially satisfied by **any** term l_1 ! For clearly, whenever

l_1 contains a negative term $\bar{l}_2$, then there exists some l_3 such that l_2 itself is the negation of l_3, namely $l_3 =_{df} \bar{l}_2$! That is, any negative term $l_1 \infty \bar{l}_2$ may superficially be transformed into a doubly-negated and hence "positive" one, $l_1 \infty \bar{\bar{l}}_3$, by simply **defining** a new term $l_3 =_{df} \bar{l}_2$ From an intuitive point of view, this trivializing construction is "incorrect" because the crucial term l_3 is **negative.** But we cannot simply modify (9) by requiring that there is some **positive** l_3 such that $l_2 \infty \bar{l}_3$, since otherwise Leibniz's definition of positiveness would become circular.

Let us therefore rather analyse Leibniz's second proposal (lines 73/74) according to which a term l_1 is positive iff any negation-operator ' - ' occurring in l_1 "is compensated (destroyed) by another ' - '".
This might be paraphrased as follows:

(10) l_1 is positive iff every occurrence of ' - ' in l_1 can be eliminated by means of the law of double negation, (1).

However, this requirement appears to be too strong. As Leibniz explained at the beginning of the essay, the negation, $\bar{l}_1$, of a positive term l_1 is a negative or privative term; and the conjunction of a positive term l_1 and a negative term $\bar{l}_2$ is a "mixed" term. These conditions apparently have to be supplemented by postulating that the conjunction of two positive terms, $l_1 l_2$, is positive while the conjunction of two negative terms, $\bar{l}_3\bar{l}_4$, is negative. Moreover, in generalization of (10), one will want to say that the negation of a negative term is a positive term. Thus in particular the negation of the (conjunctive) negative term $\overline{\bar{l}_3\bar{l}_4}$, or in other words, the **disjunction** of the two positive terms l_3 and l_4, should be regarded as positive. But, clearly, neither (1) nor any other law of Leibnitian term logic allows us to compensate the negations in '$\bar{l}_3$' or in '$\bar{l}_4$' by the negation-operator in front of their conjunction.

Actually, there is a more serious difficulty connected with (10). Leibniz would presumably accept the following condition of adequacy for any determination of the positiveness of terms:

(11) If l_1 is positive, and if $l_2 \infty l_1$, then l is positive, too.

Now the basic laws of term logic entail that l contains the **tautological** term T = 'non-(l_1 non-l_1)' and that l_1 therefore coincides with the conjunction l_1T.[6] Hence, if l_1 is positive, so should be l_1T. But the second occurrence of 'non' in T (or in l_1T) evidently is not "destroyed" by the first one, so that according to the preliminary definition (10) T and l T would **not** count as positive.

Let us suppose for a moment that we had found an improved version of (10) which not only satisfies the condition of adequacy, (11), but which also grants the disjunction of two positive terms the status of positiveness. Then Leibniz's definition of primitivy, (8), would still lead into trouble. For - on the one hand - a primitive (and hence positive) term l_1 contains the disjunction of l_1 with some other primitive term l_2 ($\not\infty$ l_1). Therefore, according to (8), l_1 would have to **coincide** with this disjunction. On the other hand, the term l_2 itself contains the same disjunction, non($\bar{l}_1 \bar{l}_2$), and thus it would contain also l_1 which coincides with the latter. Hence we would obtain by another application of (8) that $l_1 \infty l_2$ which contradicts our assumption.

If, furthermore, a modified definition of positivity would classify the tautological term T as positive, then Leibniz's definition of primitivity, (3) or (8), would entail the totally unacceptable result that T is the only primitive term. For l_1 contains the positive term T and thus, if l_1 is primitive, it would have to coincide with T. To escape these difficulties. I would like to suggest the following improvement of (3) or (8) which largely retains Leibniz's intentions:

(12) l_1 is primitive iff l_1 is positive but l_1 is not conjunctively composed of two **independent** concepts l_2, l_3, i.e. for every $\tilde{\Gamma}_2\ \tilde{\Gamma}_3$, then $\widetilde{l_2 \quad l_3}$ or $\widetilde{l_3 \quad l_2}$.

It remains an open problem, however, to find an improved version of Leibniz's definition of positiveness, (9) or (10), which avoids the aforementioned shortcomings. To be sure, the following **recursive** definition satisfies the condition of adequacy, (11), but it seems doubtful whether it is Leibnitian in spirit:

(13) a) every term letter l is positive;
b) if l is positive, then $\bar{l}$ is negative;
c) if l is negative, then $\bar{l}$ is positive;
d) if l_1 and l_2 are positive, then so is $l_1 l_2$;
e) if l_1 and l_2 are negative, then so is $\bar{l}_1\, l_2$;

f) if there is some l_2 such that l_2 is positive and $l_1 \infty l_2$, then l_1 is positive;

g) if there is some l_2 such that l_2 is negative and $l_1 \infty l_2$, then l_1 is negative.

NOTES

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** Leibniz here indicates the scope of the negation operator by drawing a line above the subsequent expression; for typographic reasons these lines have been replaced by brackets.

*** Again Leibniz's lines drawn above the formulae have been replaced by ordinary brackets.

1 Cf. in particular GI and several fragments in C. We use the standard abbreviations for Leibniz's works, i.e.:
C = L. Couturat (ed.) **Opuscules et fragments inédits de Leibniz.** Paris, 1903;
GI = F. Schupp (ed.) **Generales Inquisitiones de Analysi Notium et Veritatum.** Hamburg, 1982.

2 This "traditional" conception of privative terms must not be mixed up with Leibniz's "metaphysical" (or ontological) theory of positive, privative, and semi-privative terms as developed in C, 264-270. For a critical discussion of this theory cf. Lenzen (1989).

3 Cf. Lenzen (1984) or Lenzen (1990), chapter 3.

4 Cf. e.g. § 16 GI: " *A continet B seu (...) A coincidere ipsi BY* ". At the end of this § Leibniz formulated (apparently for the first time) the simplified law

(4) $l_1 C l_2$ iff $l_1 \infty l_1 l_2$

remarking himself: " *Notabile est pro A = BY posse etiam dici*

A = AB et ita non opus est assumtione novae literae ". The fact that in the present essay Leibniz relies on (4) rather than on (4) may be regarded as evidence for the assumption that it was written before the GI, i.e. before 1686.

5 Cf. e.g. C 262 (# 5): " *Hypothetica nihil aliud est quam categorica, vertendo antecedens in subjectum et consequens in praedicatum. Ex.gr. (...) A est B, ergo C est D. A esse B sit L, et C esse D sit M, dicemus L est M* ". A detailed discussion of this topic may be found in Lenzen (1987).

6 It is a fundamental law of conjunction that "AB est A" (C 263); in particular $1\overline{1}$ contains $\overline{1}$; hence by the principle of contraposition (e.g., GI § 77: " *Generaliter A esse B idem est quod non-B esse non-A* ") it follows that $\overline{\overline{1}}$, i.e. 1 , contains the negation of ($1\overline{1}$), i.e. T.

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*** Universität Osnabrück**