NEW TRENDS AND OPEN PROBLEMS IN FUZZY LOGIC AND APPROXIMATE REASONING

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BIBLID [ISSN 0495-4548 (1996) Vol. 11: No 27; p. 109-121]

ABSTRACT: This short paper about fuzzy set-based approximate reasoning first emphasizes the three main semantics for fuzzy sets: similarity, preference and uncertainty. The difference between truth-functional many-valued logics of vague or gradual propositions and non fully compositional calculi such as possibilistic logic (which handles uncertainty) or similarity logics is stressed. Then, potentials of fuzzy set-based reasoning methods are briefly outlined for various kinds of approximate reasoning: deductive reasoning about flexible constraints, reasoning under uncertainty and inconsistency, hypothetical reasoning, exception-tolerant plausible reasoning using generic knowledge, interpolative reasoning, and abductive reasoning (under uncertainty). Open problems are listed in the conclusion.

Keywords: Fuzzy sets, uncertainty, preference, similarity, approximate reasoning, default reasoning.

1. Introduction

Fuzzy set-based approximate reasoning was proposed and developed by Zadeh in the seventies (e.g., Zadeh, 1979). In Zadeh's approach, each granule of knowledge is expressed by means of fuzzy sets. More precisely, it is represented by a possibility distribution (computed as a combination of fuzzy set membership functions in the general case), which acts as an elastic restriction on the possible values of the variables involved in the piece of knowledge. Then, the representations of different granules of knowledge can be conjunctively combined and projected on the referentials of variables of interest in order to deduce conclusions. Finally, the possibility distribution which is thus obtained can be linguistically approximated in terms of pre-defined fuzzy sets, modifiers and connectives in order to provide the user with a conclusion expressed in an agreed upon vocabulary. If we except this last linguistic approximation step, approximate reasoning in the fuzzy set and possibility theory framework appears to be a machinery somewhat similar to what is done in the probabilistic setting where join and marginal probability distributions are computed (using other representation conventions, and a different algebraic structure). However, Zadeh has also suggested some similarity with logical deduction, by emphasizing a particular case of the combination/projection procedure, named "generalized modus ponens", where from a fact of the form "X is A'" and a rule "if X is A then Y is B" (where X and Y are variables, A', A and B are fuzzy sets), a conclusion Y is B' is computed. The generalized modus ponens can be also understood in terms of fuzzy truth-values (Zadeh, 1979; Baldwin, 1979), where the truth-value of a proposition "X is A" is viewed as its compatibility with respect to what is actually known, say, "X is A'" (this compatibility is computed as the fuzzy set of possible values of the membership $\mu_A(u)$ when the fuzzy range of u is A'). But, Zadeh's view of fuzzy logic, where "truth-values are linguistic, i.e. of the form "true", "not true", "very true", "more or less true", "false", "not very false", etc., and the rules of inference are approximate rather than exact" (as stated in (Zadeh, 1979)), departs from classical logic, and even multiple-valued logics, concerns. In the recent past years, Zadeh (1994) has particularly emphasized the key role of fuzzy rules in approximate reasoning.

As such, the approach is rather general, but on the one hand it does not say what particular types of approximate reasoning can be captured in this framework, and on the other hand, its situation with respect to logical formalisms remains unclear. This short paper intends to suggest some answers to these two questions by providing a brief overview of the research trends (for a general survey of the literature on fuzzy set-based approximate reasoning, see (Dubois and Prade, 1991)). A presentation of some fuzzy set-based logical formalisms is given in Section 3. Different semantics of fuzzy rules are discussed in Section 4, before giving a survey of the various kinds of approximate reasoning where fuzzy set-based methods apply in Section 5. Open problems are mentioned in the conclusion. In order to clarify what kinds of practical problems we may have in mind when dealing with approximate reasoning, the three main potential semantics of fuzzy sets, namely, similarity, preference and uncertainty, are first emphasized in Section 2 and the crucial difference between degree of truth and degree of uncertainty is stressed.

2. The Semantics of Fuzzy Sets - Fuzziness Versus Uncertainty

Fuzzy sets seem to be relevant in three classes of applications where approximate reasoning plays a role: classification and data analysis, decision-making problems, and reasoning under uncertainty. These three directions, that have been investigated by many researchers, actually correspond and/or exploit three semantics of the membership grades, respectively in terms of similarity, preference and uncertainty. Indeed, considering the degree of membership $\mu_F(u)$ of an element u in a fuzzy set F, defined on a referential U, one can find in the literature, three interpretations of this degree:

- degree of similarity: $\mu_F(u)$ is the degree of proximity of u to prototype elements of F. Historically, this is the oldest semantics of membership grades (Bellman, Kalaba and Zadeh, 1966). This view is particularly suitable in classification, clustering, regression analysis and the like, where the problem is that of abstraction from a set of data. It is also at work in fuzzy control techniques, where the similarity degrees between the current situation and the prototypical ones described in the condition parts of the rules, are the basis for the interpolation mechanism between the conclusions of the rules;

- degree of preference: F represents a set of more or less preferred objects (or values of a decision variable x) and $\mu_F(u)$ represents an intensity of preference in favor of object u, or the feasibility of selecting u as a value of x. This view is the one later put forward by Bellman and Zadeh (1970); it has given birth to an abundant literature on fuzzy optimisation and decision analysis. Approximate reasoning is then concerned with the propagation of preferences when several constraints (which may be fuzzy) relate the variables. Applications pertain to design and scheduling problems;
- degree of uncertainty: this interpretation was proposed by Zadeh (1978) when he introduced possibility theory. $\mu_F(u)$ is then the degree of possibility that a parameter x has value u, given that all that is known about it is that "x is F". Possibility can convey an epistemic meaning (F then describes the more or less plausible values of x) or a physical meaning ($\mu_F(u)$ being the degree of ease of having x = u). Viewing $\mu_F(u)$ as a degree of uncertainty only refers to the epistemic interpretation. The physical meaning of possibility has more to do with preference and feasibility.

There always remains some confusion in the fuzzy literature on the potential of fuzzy sets for handling uncertainty. This state of confusion can be exemplified in some texts by fuzzy set proponents claiming that probability theory models randomness while fuzzy set models subjective uncertainty (hence ignoring subjective probability). It is also present in the expert systems literature where certainty factors have been confused with membership grades. It also pervades the antifuzzy literature where the truth-functionality of conjunction, disjunction and negation in fuzzy logic is considered as mathematically inconsistent (see (Elkan, 1994) for a recent restatement of this fallacy).

In fact, insofar as fuzzy sets model vague predicates, membership grades model degrees of truth of vague propositions, not degrees of uncertainty. In the scope of knowledge representation, truth is a matter of convention, while uncertainty reflects incomplete or contradictory knowledge. In classical logic the convention is that truth is binary. Fuzzy set theory (and before, multivalued logics) has modified this convention. This shift in convention does not entitle degrees of truth to be interpreted as degrees of uncertainty.

Degrees of uncertainty can be attached to a non-fuzzy proposition in order to model the fact that it is not known whether this proposition is true or false. Uncertainty is at the meta-level with respect to truth. In classical logic truth is binary (true or false) while uncertainty is ternary (surely true, surely false, or unknown). In probability theory, truth is usually binary (crisp propositions) while uncertainty takes on all values in the unit interval. In fuzzy set theory truth is many-valued but there is no uncertainty insofar as the element, the membership grade of which is computed, is precisely located. Knowing that a bottle is precisely half full, we can say that it is half true that the bottle is full, which does not mean at all that the probability that the bottle is full is one half (in this latter case, it is possible that the bottle is in fact empty for instance). Fuzzy-truth values (which Zadeh has claimed to be typical of fuzzy logic) are possibility distributions that describe partially unknown truth-values.

As pointed out above, a degree of membership $\mu_F(u)$ is interpreted as a degree of truth or as a degree of uncertainty depending on the cases. When it is a degree of truth it is attached to the fuzzy proposition 'X is F' and it is known that X = u. As a degree of uncertainty, $\mu_F(u)$ is attached to the non-fuzzy proposition X = u, when all that is known is that the value of X is somewhere in the support of F; then $\mu_F(u)$ is interpreted as the degree of possibility that X = u.

3. Logical Embeddings

Very different extensions of classical logic that exploit the notion of a fuzzy set have been proposed. Some are truth-functional, while others are not. We may distinguish between:

- *many-valued logics* that were proposed before fuzzy set theory came to light. They are exclusively devoted to the handling of "vague" propositions $\tilde{p}$, i.e., propositions which may be partially true (e.g., propositions involving properties whose satisfaction is a matter of degree). The underlying algebraic structure is then weaker than a Boolean algebra, and can be consistent with truth-values $t(\tilde{p})$ that lie in the unit interval and remain truth-functional. See, e.g., (Hajek, 1995) for an introduction.
- *possibilistic logic* that is built on top of classical logic, and where each crisp proposition is attached a lower bound of a degree of necessity N(p) expressing the certainty of p given the available information. N(p) = 1 iff p is surely true and N(p) = 0 expresses the complete lack of certainty that p is true (either p is false and then $N(\neg p) = 1$, or it is unknown if p is true or false and then $N(\neg p) = 0$). The degree N(p) is compositional *for conjunction only* $n(N(p \wedge q) = \min(N(p),N(q))$, and $N(p \vee q) \geq \max(N(p),N(q))$ generally). For instance, if $q = \neg p$, $p \vee q$ is tautological, hence surely true ($N(p \vee q) = 1$), but p may be unknown ($N(p) = N(\neg p) = 0$). Moreover $N(\neg p) = 1 - \Pi(p)$ where $\Pi(p)$ is the degree of possibility of proposition p. Functions N and Π stem from the existence of a fuzzy set of more or less possible worlds, one of which is the actual one. It is described by means of a possibility distribution π on the interpretations of the language, and $N(p) = 1 - \sup\{\pi(\omega), \omega \models \neg p\} = 1 - \Pi(\neg p)$, i.e., N(p) is computed as the degree of impossibility of the proposition $\neg p$. A possibilistic logic formula (p,α) understood as $N(p) \geq \alpha$, is represented by a fuzzy set such that interpretations which make p false have degree $1 - \alpha \geq \Pi(\neg p)$ (i.e., the possibility that p is false is upper bounded by $1 - \alpha$), while the interpretations which make p true have degree 1. Then, the possibility distribution π representing a set of possibilistic formulas is obtained by the min-conjunction of the fuzzy sets representing the formulas (Dubois, Lang and Prade, 1994).

In possibilistic logic a fuzzy set describes incomplete knowledge about where the actual world is, while in many-valued logics fuzzy sets describe the extensions of vague predicates. The truth-functionality is not compulsory however when dealing with vagueness:

In *similarity logics* it is supposed that the vagueness of predicates stems from a closeness relation (Ruspini, 1991) that equips the set of interpretations of the language. Then a Boolean proposition p is actually understood as a fuzzy proposition $\tilde{p}$ whose models are *close* to models of p. Let [p] be the set of models of p and $R(p) = [p] \hat{o} R$ the fuzzy set of models close (in the sense of fuzzy relation R) to models of p. Then generally, $R(p \wedge q) \subseteq R(p) \cap R(q)$, without equality, so that truth-values are not truth-functional with respect to conjunction.

In classical logic, a proposition p entails another proposition q whenever each situation where p is true is a situation where q is true. Entailment is denoted $\models$, and $p \models q$ means $[p] \subseteq [q]$ where [p] is the set of models of p.

In possibilistic logic, the type of inference which is at work is plausible inference. The possibility distribution π on the set of interpretations encodes an ordering relation that ranks possible worlds ω in terms of plausibility. Then p plausibly entails q if and only if q is true in the most plausible situations where p is true, i.e., $\max[p] \subseteq [q]$, where $\max[p] = \{\omega \in [p], \pi(\omega) \text{ maximal}\}$. This type of inference is also called "preferential inference" in nonmonotonic reasoning. In contrast, inference in similarity logics is dual to preferential inference. The set of interpretations of the language is equipped with a similarity relation R. Then p entails approximately q in similarity logic if all the situations where p is true are close to situations where q is true, i.e., $[p] \subseteq \text{support}[R(q)]$. More generally, a degree of strength of the entailment can be computed as $I(q \mid p) = \inf_{\omega \in [p]} \sup_{\omega' \in [q]} \mu_R(\omega,\omega')$. It plays in similarity logic the same role as a degree of confirmation in inductive logic. In possibilistic logic the counterpart of $I(q \mid p)$ is the conditional necessity $N(q \mid p)$ computed from π, i.e., $N(q \mid p) = N(\neg p \vee q) > 0$ if $\Pi(q \wedge p) > \Pi(\neg q \wedge p)$, and $N(q \mid p) = 0$ otherwise.

These newly emerged notions of fuzzy set-based inference certainly deserve further developments, and lead to very different types of logic. Of interest is the study of their links to the usual entailment principle of Zadeh (defined as a fuzzy set inclusion), and various extensions of consequence relations in multiple-valued logic, as studied by Chakraborty (1988), Castro et al. (1994).

4. Fuzzy Rules

As already said, in possibilistic or in similarity logics, the propositions are not fuzzy by themselves, although possibilistic propositions, or classical propositions blurred by similarity are represented by fuzzy sets. Fuzzy rules involve fuzzy sets directly in their expression. Strangely enough, in spite of the acknowledged importance of fuzzy rules and of the great number of works in fuzzy set-based approximate reasoning, there has been very little concern until now about the possible intended meanings of a fuzzy rule, although fuzzy rules seem to play an important role in our thinking mechanisms. However, several kinds of fuzzy rules can be distinguished from a semantic point of view (Dubois and Prade, 1991, 1995).

- *Certainty rules:* they are of the form "the more x is A, the more certain y lies in B". The rule is interpreted as "if x = u, it is at least $\mu_A(u)$-certain that y is B", which is represented by a joint possibility distribution $\pi_{x,y}(u,v)$ upper bounded by $\max(1 - \mu_A(u), \mu_B(v))$, since values outside B are at most $(1 - \mu_A(u))$-possible for y;
- *Gradual rules:* they are of the form "the more x is A, the more y is B". Their representation is such that $\pi_{x,y}(u,v) = 1$ if $\mu_A(u) \leq \mu_B(v)$. When $\mu_A(u) > \mu_B(v)$, $\pi_{x,y}(u,v)$ can be taken to be equal to 0 or $\mu_B(v)$ depending if the relation between x and y is fuzzy or not;
- *Possibility rules:* they are of the form "the more x is A, the more possible y is B", with the intended meaning that if x = u, *any* value in B is at least $\mu_A(u)$-possible. Their representation $\pi_{x,y}(u,v)$ is thus lower-bounded by $\min(\mu_B(v), \mu_A(u))$. This corresponds to the modelling of the rules in Mamdani's method for fuzzy control;
- *Antigradual rules*: they express that "the more x is A (i.e., the greater $\mu_A(u)$ if x = u), the larger the set of values which are possible for y" (for instance, we may take for y the set $\{v \mid \mu_B(v) > 1 - \mu_A(u)\}$).

It is important to notice that certainty and gradual rules contrast with possibility and antigradual rules. The conclusions of the former have to be combined conjunctively since they assess *restrictions* on possible values, while the conclusions of the latter have to be combined disjunctively since they express sets of values which are just *possible*.

5. Various Kinds of Approximate Reasoning

Fuzzy sets offer a powerful tool for the modelling of various kinds of commonsense reasoning, where the three semantics of membership functions interfere. Let us review some of them.

Fuzzy deductive inference

This type of approximate reasoning has been advocated by Zadeh in the mid-seventies, as a calculus of fuzzy restrictions. The principles of this approach were recalled in the introduction. They rely on the conjunctive combination of possibility distributions and their projection on suitable subspaces. This conjunction/projection method is at work in the POSSINFER system of Kruse et al. (1994). This type of inference is also a generalization of constraint propagation to flexible constraints, provided that one interprets each statement as a requirement that some controllable variable must satisfy. Then the possibility distributions model preference, and inference comes down to consistency analysis (such as arc-consistency, path-consistency, etc.) in the terminology of constraint-directed reasoning. The advantage of fuzzy deductive inference is to directly account for flexible constraints and prioritized constraints, where the priorities are modelled by means of necessity functions (see Dubois, Fargier and Prade, 1994).

Reasoning under uncertainty and inconsistency

A possibilistic knowledge base K is a set of pairs (p, s) where p is a classical logic formula and s is a lower bound of a degree of necessity ($N(p) \geq s$). It can be viewed as a stratified deductive data base where the higher s, the safer the piece of knowledge p. Reasoning from K means using the safest part of K to make inference, whenever possible. Denoting $K_\alpha = \{p, (p,s) \in K, s \geq \alpha\}$, the entailment $K \vdash (p,\alpha)$ means that $K_\alpha \vdash p$. K can be inconsistent and its inconsistency degree is $inc(K) = \sup\{\alpha, K \vdash (\bot,\alpha)\}$ where $\bot$ denotes the contradiction. In contrast with classical logic, inference in the presence of inconsistency becomes non-trivial. This is the case when $K \vdash (p,\alpha)$ where $\alpha > inc(K)$. Then it means that p follows from a consistent and safe part of K (at least at level α). This kind of syntactic non-trivial inference is sound and complete with respect to the above defined preferential entailment. Moreover adding p to K and nontrivially entailing q from $K \cup \{p\}$ corresponds to revising K upon learning p, and having q as a consequence of the revised knowledge base. This notion of revision is exactly the one studied by Gärdenfors (1988) at the axiomatic level.

Nonmonotonic plausible inference using generic knowledge

Possibilistic logic does not allow for a direct encoding of pieces of generic knowledge such as "birds fly". However, it provides a target language in which plausible inference from generic knowledge can be achieved in the face of incomplete evidence. In possibility theory "p generally entails q" is understood as "$p \wedge q$ is a more plausible situation than $p \wedge \neg q$". It defines a constraint of the form $\Pi(p \wedge q) > \Pi(p \wedge \neg q)$ that restricts a set of possibility distributions. Given a set S of generic knowledge statements of the form "p_i generally entails q_i", a possibilistic base can be computed as follows. For each interpretation ω of the language, the maximal possibility degree $\pi(\omega)$ is computed, that obeys the set of contraints in S. This is done by virtue of the principle of minimal specificity (or commitment) that assumes each situation as a possible one insofar as it has not been ruled out. Then each generic statement is turned into a material implication $\neg p_i \vee q_i$, to which $N(\neg p_i \vee q_i)$ is attached. It comes down, as shown in Benferhat et al. (1992) to rank-ordering the generic rules giving priority to the most specific ones, as done in Pearl (1990)'s system Z. A very important property of this approach is that it is exception-tolerant. It offers a convenient framework for implementing a basic form of nonmonotonic system called "rational closure" (Lehmann and Magidor, 1992), and addresses a basic problem in the expert system literature, that is, handling exceptions in uncertain rules.

Hypothetical reasoning

The idea is to cope with incomplete information by explicitly handling assumptions under which conclusions can be derived. To this end some literals in the language are distinguished as being asssumptions. Possibilistic logic offers a tool for reasoning with assumptions. It is based on the fact that in possibilistic logic a clause $(\neg h \vee q, \alpha)$ is semantically equivalent to the formula with a symbolic weight (q, min (α, t(h)) where t(h) is the (possibly unknown) truth value of h. The set of environments in which a proposition p is true can thus be calculated by putting all assumptions in the weight slots, carrying out possibilistic inference so as to derive p. The subsets of assumptions under which p is true with more or less certainty can be retrieved from the weight attached to p. This technique can be used to detect minimal inconsistent subsets of a propositional knowledge base (see Benferhat et al., 1994).

Interpolative Reasoning

This type of reasoning is at work in fuzzy control applications, albeit without clear logical foundations. Klawonn and Kruse (1993) have shown that a set of fuzzy rules can be viewed as a set of crisp rules along with a set of similarity relations. Moreover an interpolation-dedicated fuzzy rule 'if is A then Y is B" can be understood as "the more x is A the more Y is B" and the corresponding inference means that if $X = x$ and $\alpha = \mu_A(x)$ then Y lies in the level cut B_α. When two rules are at work, such that $\alpha_1 = \mu_{A_1}(x)$, $\alpha_2 = \mu_{A_2}(x)$, then the conclusion $Y \in (B_1)_{\alpha_1} \cap\pi\ (B_2)_{\alpha_2}$ lies between the cores of B_1 and B_2, i.e., on ordered universes, an interpolation effect is obtained. It can be proved that Sugeno's fuzzy reasoning method for control can be cast in this framework (Dubois, Grabisch, Prade, 1994). More generally interpolation is clearly a kind of reasoning based on similarity (rather than uncertainty) and it should be related to current research on similarity logics (Dubois et al., 1995). More generally similarity relations and fuzzy interpolation methods should impact on current research in case-based reasoning.

Abductive reasoning

Abductive reasoning is viewed as the task of retrieving plausible explanations of available observations on the basis of causal knowledge. In fuzzy set theory causal knowledge has often been represented by means of fuzzy relations relating a set of causes C to a set of observations S. However the problem of the semantics of this relation has often been overlooked. $\mu_R(c,s)$ may be viewed either as a degree of intensity or a degree of uncertainty. Namely, when observations are not binary, $\mu_R(c,s)$ can be understood as the intensity of presence of observed symptom s when the cause c is present. This is the traditional view in fuzzy set theory. It leads to Sanchez (1977) approach to abduction, based on fuzzy relational equations. Another view has been recently proposed by the authors, where by $\mu_R(c,s)$ is understood as the degree of certainty that a binary symptom s is present when c is present. A dual causal matrix R' must be used where $\mu_{R'}(c,s)$ is the degree of certainty that a binary symptom s is absent when c is present. On such a basis the theory of parsimonious covering for causal diagnosis by Peng and Reggia (1990) can be extended to the case of uncertain causal knowledge and incomplete observations. This method is currently applied to satellite failure diagnosis (Cayrac et al., 1994).

6. Conclusion and Open Problems

Fuzzy sets open a lot of new areas of research, or at least a new way of investigating old questions in a unified way. They are useful to model vague predicates in reasoning, but also for coping with incomplete information. Fuzzy sets are also a natural framework for reasoning with similarity and preference. A number of open problems remains, some of which are listed hereafter:

A syntax for fuzzy logic

A lot of works have been done in fuzzy logic at the semantic level, but few papers address the question of finding a proper syntax. Most of these papers (e.g., Pavelka, 1979) put intermediary truth-values in the object language. The only well-known axiomatic system is the one for Lukasiewicz logic, that relies on MV-algebras. But Zadeh's fuzzy logic is not based on MV-algebra. Offering a syntax to Zadeh's fuzzy logic may cancel the main objection raised by logicians against fuzzy set theory.

Refining the min-based ordering

Fuzzy constraint satisfaction is based on the idea of finding a solution that best satisfies the most violated constraint. A lot of solutions may remain indiscriminated in such a process. The question is how to refine this ordering while remaining faithful to that egalitarian principle. Several proposals are being studied that derive from set-inclusion and lexicographic methods borrowed from nonmonotonic reasoning and social sciences (e.g., Dubois, Fargier and Prade, 1995).

Implementing nonmonotonic reasoning systems in possibilistic logic

So far only rational and preferential inference have been captured. Some other forms of nonmonotonic reasoning approaches such as circumscription (Benferhat et al., 1993) and default theory (Yager, 1987) should be investigated with this purpose in view.

Expressing and handling independence statements in possibility theory

Commonsense exception-tolerant reasoning relies not only on generic rules, but also on independence assumptions. In logic conditional independence of propositional variables is implicitly assumed due to the monotonicity of the inference. On the contrary, independence must be explicitly postulated in probability theory. This topic deserves a lot of investigations in possibility theory because several concepts of independence look reasonable. Preliminary results can be found in (Dubois et al., 1994; Fonck, 1994; De Campos et al., 1995)

Defining a full-fledged logic of similarity

The purpose is to handle approximate reasoning problems and account for interpolation. Especially it might be useful to cast both similarity logic and possibilistic logic in the same setting so as to come up with a system of approximate inference under incomplete information; see Esteva et al. (1994) for a preliminary work.

Fuzzy abduction with prior knowledge

All fuzzy relational abductive reasoning methods neglect the fact that the retrieval of causes is partially based on prior knowledge especially when obervations are scare.

Bayesian methods of diagnosis exploit prior knowledge. How to account for prior knowledge using fuzzy methods?

Fuzzy sets with non-numerical scales

Another point is that although fuzzy sets have been introduced with a numerical flavor, a membership function is not necessarily mapped on a set of numbers, but an ordered set such as a complete lattice is enough. A membership function can even be construed as an ordering relation $\geq_F$, attached to a predicate F, where u $\geq_F$ u' means that u is more F than u'. A fuzzy relation R on U × U can also be viewed as a ternary relation (on U^3), i.e., a collection $\{\geq_u, u \in U\}$ of binary relations that are complete preorderings. Then $\mu_R(u,u') \geq \mu_R(u,u'')$ can represent a situation where u' $\geq_u$ u", which reads u' is closer to u than u". These structures are common in conditional logics of counterfactuals (Lewis, 1973) to which similarity logics can be related.

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