On the strategic choice of spatial price policy: the role of the pricing game rules

February 2001

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Abstract

In this paper, we show that the strategic choice of spatial price policy under duopoly crucially depends on the rules of price competition. Thisse and Vives (1988) show that spatial price discrimination is a dominant strategy when the mill pricing firm is the leader and the discriminatory firm is the follower. When the leader-follower roles are reversed we find that equilibrium pricing policies depend on the consumer´s reservation value. The pricing policy game has two equilibria in pure strategies, either both firms price uniformly (f.o.b.) or both firms price discriminate, when the reservation value is low. For intermediate levels of the reservation value, price discrimination is a dominant strategy and the pricing policy game is similar to a Prisoner´s Dilemma. When the consumer reservation value is large enough we obtain asymmetric equilibria in which one firm prices according to f.o.b. and the other price discriminates. We also analyze the case of simultaneous price competition and find a mixed strategies equilibrium for the price competition subgame such that the pricing policy game has two equilibria in pure strategies, either both firms price uniformly or both firms price discriminate.

Key words: spatial price discrimination, pricing policy.
JEL Classification: L12, R32.

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Financial support from DGES (PB97-0603), UPV (035.321-HB159/98) and GV (PI-1998-86) is gratefully acknowledged.

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1.- **Introduction**

There are two general spatial pricing policies: mill pricing (or f.o.b) and delivery pricing. If the pricing policy is f.o.b. (free on board) consumers pick up the product at the mill, paying the mill price and incurring the freight cost. Delivered pricing policies are pricing rules not based on consumers´ picking up the product at the mill. The most common delivered pricing rules are basing-point pricing and uniform delivered pricing. In a uniform delivered pricing system each firm quotes the same price to all consumers, regardless of distance. In a basing-point pricing system, firms decide on the location of a base point and a price at that location (the base price); the price at any other location is calculated as the base price plus transportation charges from the base point.[1] We define a delivered pricing rule as any price function other than f.o.b. Note that a delivered pricing policy entails spatial price discrimination.[2]

The existence of non-negligible transportation costs can also be interpreted in terms of product differentiation.[3] In this context, f.o.b. pricing corresponds to a firm producing a single variety of the good and the consumer having to adapt the product to his preferences (transportation costs represent the utility loss for not consuming the preferred variety). A delivered price schedule corresponds to a firm producing several varieties of the product and being able to price discriminate among consumers (sell the

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[1] In some markets delivered pricing policies have been widely used. Examples of basing-point pricing policies are the Pittsburgh Plus system used in the steel industry and the Portland Plus system used for plywood. See Machlup (1949), Scherer (1980) and Phlips (1983).

[2] There is price discrimination whenever the difference in the end price at any two locations does not fully reflect the differences in transportation costs; in other words, when the net price (delivered price minus freight costs) is not constant.

[3] Hotelling (1929).

different varieties at prices that do not reflect the different transportation costs).

Thisse and Vives (1988) analyze the strategic choice of spatial pricing policy in a duopoly market with homogeneous product and inelastic demand; they conclude that f.o.b. is not an equilibrium pricing system and firms will choose discriminating pricing policies. In fact, a typical Prisioner´s Dilemma arises since price discrimination is a dominant strategy but firms would make more profits under f.o.b. pricing. Their result also holds when the circular model of product differentiation is considered.

Eber (1997) investigates the robustness of this result by also considering the choice of location. He shows that discriminatory pricing is the unique equilibrium outcome (in dominant strategies) of a three-stage sequential game in which firms choose first a location, second the price policy and, finally, a price schedule. However, when firms choose their price policy before their location, mill pricing emerges as the unique equilibrium outcome. De Fraja and Norman (1993) obtain an asymmetric equilibrium in which one firm prices according to f.o.b. and the other price discriminates in a model with differentiated goods and elastic demand.

All the above works share certain assumptions on the rules of the pricing game. Two types of price competition are considered: if both firms have chosen the same pricing policy in the previous stage, firms decide price levels simultaneously and independently. When the two firms choose different pricing policies, the mill pricing firm becomes a price leader and the discriminatory firm is a follower reacting optimally to the mill price. The argument for this change in the rules of the game is that when firms choose different pricing policies there is no equilibrium in pure strategies in the pricing game. In this paper, we show that the result that discriminatory pricing is a dominant strategy depends crucially on this change in the rules of the game.

Firstly, we show that if the leader and follower roles are reversed so the discriminatory firm becomes the price-leader, the pricing policy game is a Prisoner´s Dilemma only for intermediate reservation values. When the reservation value is low, the pricing policy game is not a Prisoner´s Dilemma and, in fact, two Nash equilibria in pure strategies arise in which both firms choose the same policy, that is both firms engage in mill pricing or both firms price discriminate. When the consumer reservation value is large enough we obtain two asymmetric equilibria in which one firm prices according to f.o.b. and the other price discriminates. It must be stressed that the last case is the most relevant given the critical levels of the reservation value.

We next consider the problem when all the price subgames are played under the same rules: simultaneous price competition. This approach requires us to solve the asymmetric (different pricing policies) price subgames allowing mixed strategies. Note that in these subgames the mill pricing firm charges one price and the discriminatory firm can charge an infinite number of different prices. As a consequence, the strategy spaces of the two firms have different dimension. Therefore, such a game not only exhibits discontinuities in the payoffs but also has infinite strategy spaces of different dimension for each player. We shall show that there is an equilibrium in which the mill pricing firm follows a mixed strategy whereas the discriminatory firm uses a pure strategy. The pricing policy game is not a Prisoner´s Dilemma and, in fact, two Nash equilibria in pure strategies arise in which both firms choose the same policy, that is both firms engage in mill pricing or both firms price discriminate.

The paper is organized as follows. Section 2 gives a description of the model. In section 3 we describe the problem of the strategic choice of spatial pricing policy and present some results of earlier work. Sections 4 and 5 characterize the equilibria of the pricing

policy game under different modes of price competition and state the main results. Section 6 offers concluding comments.

2.- **The model**

Consumers are distributed uniformly along the unit interval [0, 1]. The location of a consumer is denoted by x and defined as the distance to the left endpoint of the market. The preferences are as follows: each consumer has a reservation value, R, for the good, and buys precisely one unit per period of time, from the firm that has the lowest end (delivered) price, as long as his total payment does not exceed his reservation value, and buys nothing otherwise. When several firms have the same delivered price at a given location the consumer chooses the supplier with the lowest transportation cost.[4] The good cannot be stored.

There are two firms, firm 1 and firm 2, that may produce a homogeneous good in the spatial market [0,1]. Firm 1 is located at the left endpoint of the market, and firm 2 at the right endpoint. Marginal costs of production are constant and identical for both firms; for the sake of notational simplicity prices are expressed net of marginal cost.

The cost of transporting one unit of the good is given by the function $t(d) = t\,d$, where d is the distance from the location of the consumer to the producer. We will assume that $R > t$.[5] The delivered price at a location x must cover the total (production plus transport) marginal cost. If firm i were to price below total marginal cost it could do at

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[4] The assumption that price ties are broken in the socially efficient way is fairly standard in literature. See, for example, Lederer and Hurter (1986) for a justification.

[5] This assumption guarantees that the whole market will be served regardless of the firms´ pricing policies.

least as well, for any given price of the rival, by pricing at marginal cost.[6]

The timing of the game is as follows:

Stage 1.- Firms choose their pricing policy simultaneously and independently. That is, they decide on whether to have an f.o.b. policy or a delivered pricing policy.

Stage 2.- Each firm observes the other's pricing policy and they decide their price levels simultaneously and independently, if both firms choose the same policy. When firms choose different pricing policies, we consider three kinds of price competition: simultaneous, the mill pricing firm as leader (Thisse and Vives, 1988) and the mill pricing firm as follower.

3. **The choice of price policy**

We solve the game by backward induction to obtain the subgame perfect equilibria.

Second stage

There are several cases depending on the outcome of the previous stage:

a) Both firms price according to f.o.b.

b) Both firms use delivered pricing.

c) One firm is committed to f.o.b. and the other firm uses delivered pricing.

a) Both firms price according to f.o.b.

If both firms have chosen f.o.b. policies, they will select mill prices simultaneously and independently. The demand for each firm is given by:

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[6] See Lederer and Hurter (1986) and Thisse and Vives (1988).

$$D_i(p_i, p_j) = \left\{ \begin{array}{cc} 1 & \text{if } p_i \leq p_j - t \\ \frac{1}{2} + \frac{p_j - p_i}{2t} & \text{if } p_j + t > p_i > p_j - t \\ 0 & \text{if } p_i \geq p_j + t \end{array} \right\} \text{i, j = 1, 2, j} \neq \text{i.}$$

The profit functions are $\Pi_i(p_i, p_j) = p_i D_i(p_i, p_j)$, $i, j = 1, 2, j \neq i$. These profit functions are quasi-concave, ensuring the existence of a price equilibrium.[7] The equilibrium mill prices are given by (see figure 1): $p_1^U = p_2^U = t$. The equilibrium profits are given by $\Pi_1^{UU} = \Pi_2^{UU} = \frac{t}{2}$.

b) Both firms use delivered pricing

Denote as $p_1(x)$ and $p_2(x)$ the delivered prices of firm 1 and firm 2, respectively, at location x, $0 \leq x \leq 1$. At a given location x, competition is *à la* Bertrand: with cost asymmetries if $x \neq \frac{1}{2}$ and with the same cost if $x = \frac{1}{2}$. When $x < \frac{1}{2}$, firm1's cost is lower than firm 2's. The opposite is true when $x > \frac{1}{2}$.[8] This implies that in equilibrium the delivered price at x will equal the transportation cost of the firm located further from x. Given the previous argument,[9] the equilibrium pricing policies are given by: $p_1(x) = p_2(x) = \max\{tx, t(1 - x)\}$ for all $x \in [0, 1]$.

$$\Pi_1^{DD} = \int_0^{\frac{1}{2}} \{t(1 - x) - tx\} dx = \frac{t}{4}$$

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[7] See d'Aspremont, *et al.* (1979).

[8] When firms price according to f.o.b., they are competing in the entire market with only one strategic variable: the mill price. However, under discriminatory pricing, firms compete at each location x separately. In this situation, stability of price competition is less difficult than under f.o.b..

[9] See Lederer and Hurter (1986) for a formal proof.

$$\Pi_2^{DD} = \int_{\frac{1}{2}}^{1} \{ tx - t(1 - x)\} dx = \frac{t}{4}$$

In firm 1's market area, the end price decreases with the distance to the firm, whereas the transportation costs increase with that distance: the net price is not constant. In firm 2's market area, the net price also varies with distance and there is price discrimination.

c) One firm is committed to f.o.b. and the other firm uses delivered pricing

Simultaneous price competition

As noticed by Thisse and Vives (1988), there may not be a simultaneous move Nash equilibrium in pure strategies. Assume that the mill pricing firm (firm 1) charges a mill price p_1, then the best reply of the discriminatory firm (firm 2) is to set a pricing policy $p_2(x) = \max\{t\ (1 - x),\ p_1 + tx\}$. That is, given the price of firm 1, p_1, then the best response of firm 2 is to equal the corresponding full price whenever possible.[10] But note that if firm 2 has a pricing policy $p_2(x) = \max\{t\ (1 - x),\ p_1 + tx\}$, the best response of the mill pricing firm is to sell to consumers ε-below p_1 in order to capture the whole market. For any p_1, given the best response of the other firm, the mill pricing firm always has the incentive to reduce its price slightly and sell to the entire market. In section 5 we solve the pricing policy game by allowing mixed strategies in the pricing game.

We next analyze the different pricing policy subgames under the three rules specified above. We start by assuming, as per Thisse and Vives (1988), that the mill pricing firm is the price leader and the discriminatory firm is the follower.

[10] Recall that according to our assumption if the two firms quote the same price consumers buy from the firm with lower total delivered cost.

The mill pricing firm is the leader

Assume that the mill pricing firm is firm 1 and the discriminatory firm is firm 2. Given firm 1´s price p_1 the market boundary $\tilde{x}$ is determined by $p_1 + t\tilde{x} = t(1 - \tilde{x})$, which yields $\tilde{x} = (t - p_1)/2t$, since the optimal response of firm 2 is to match firm 1´s full price $p_1 + t\tilde{x}$, whenever possible, that is when $p_1 + t\tilde{x} \geq t(1 - \tilde{x})$. Profits of firm 1 are given by $\Pi_1 = p_1\tilde{x}$ and the optimal price for firm 1 is $p_1^* = t/2$ with associated market boundary $\tilde{x}^* = 1/4$, yielding profits of $\Pi_1^* = \Pi_1^{UD} = t/8$. The equilibrium price schedule of firm 2 is $p_2^*(x) = \max\{p_1^* + tx, t(1 - x)\}$ and the equilibrium profits are

$$\Pi_2^* = \Pi_2^{UD} = \int_{\frac{1}{2}}^{1} \{\frac{t}{2} + tx - t(1 - x)\}dx = \frac{9t}{16}$$

When firm 2 is the mill pricing firm and firm 1 the discriminatory firm we obtain the symmetric results, thus, $\Pi_1^{UD} = \Pi_2^{DU}$ and $\Pi_1^{DU} = \Pi_2^{UD}$. Given these profits, Table 1 summarizes the possible outcomes of the second stage. Note that we obtain the typical Prisioner´s Dilemma since $\Pi_1^{DU} > \Pi_1^{UU} > \Pi_1^{DD} > \Pi_1^{UD}$ and $\Pi_2^{UD} > \Pi_2^{UU} > \Pi_2^{DD} > \Pi_2^{DU}$. That is, price discrimination is a dominant strategy, although firms would be better under mill pricing. The general conclusion of Thisse and Vives (1988) is that there is a robust tendency for a firm to choose the discriminatory policy. However, we show that their result crucially depends on the rules of price competition.

4. **The discriminatory firm is the price-leader**

In order to provide intuition on the equilibrium outcome, we derive the equilibrium by construction. Suppose that firm 2, anticipating that the follower may undercut its price in order to capture the whole market, decides to implement a pricing policy of $p_2(x) = \max\{t(1 - x), tx\}$. Given this pricing policy, firm 1 would react by setting a price $p_1 = t$

/2, and firms profits would be $\Pi_1 = t/8$ and $\Pi_2 = \int_{\frac{1}{2}}^{1} \{tx - t(1 - x)\}dx = \frac{t}{4}$. However, firm 2 might increase profits by increasing its full price up to (or to just below) $p_2'(x) = \max\{t(1 - x), t/8 + tx\}$. Note that firm 1 would be (almost) indifferent between charging a price $t/2$ or charging a price $t/8 - \varepsilon$ and capturing the whole market. Profits would be $\Pi_1 = t/8$ and $\Pi_2' = \int_{\frac{1}{4}}^{1} \{\frac{t}{8} + tx - t(1 - x)\}dx = \frac{81t}{256}$.

In more general terms it can be demonstrated that

Lemma 1.- The best policy for the discriminatory firm is to keep firm 1 indifferent between prices $p_1 \in [p_L, p_H]$ (or just prefer p_H), with an associated profit of $\overline{\Pi}$ for firm 1.[11]

The lower extreme of the interval, p_L, is the highest price that allows firm 1 to capture the whole market and to obtain a profit $\overline{\Pi}$ (note that $p_L = \overline{\Pi}$). This price therefore dominates prices $p_1 < p_L$. The upper extreme of the interval, p_H, is the highest price that allows firm 1 to obtain a profit $\overline{\Pi}$ given the rival´s pricing policy and the consumer reservation value.

Proof. Consider any two possible prices for the mill pricing firm: p', $p'' \in [p_L, p_H]$, with $p' \neq p''$. Assume that, given the pricing policy of firm 2, $\Pi_1(p') = \overline{\Pi} > \Pi_1(p'')$, that is firm 1 strictly prefers p' to p''. Given firm 2´s pricing policy, $p_2(x)$ and firm 1´s mill price p', the market boundary $\tilde{x}(p')$ is determined by $p' + t\,\tilde{x}(p') = p_2(\tilde{x}(p'))$, and therefore:

[11] A similar idea arises in the paper of Prescott and Visscher (1977). These authors analize sequential location among firms and show that when an incumbent firm may choose to locate two outlets in the market the optimal strategy is to keep the potential entrant indifferent as regards location in the market.

$$\Pi_1(p') = \overline{\Pi} = p'[\frac{p_2(\tilde{x}(p')) - p'}{t}] > p''[\frac{p_2(\tilde{x}(p'')) - p''}{t}] = \Pi_1(p'')$$

As a consequence, firm 2 could increase its price at $\tilde{x}(p'')$ with no loss of market share and could increase profits. Q.E.D.

We next derive the optimal pricing policy for the discriminatory firm. Given firm 2´s pricing policy, $p_2(x)$, and firm 1´s mill price, p_1, the market boundary $\tilde{x}(p_1)$ is determined by $p_1 + t\ \tilde{x}(p_1) = p_2(\tilde{x}(p_1))$, which yields

$$\tilde{x}(p_1) = \frac{p_2(\tilde{x}(p_1)) - p_1}{t} \qquad (1)$$

To keep firm 1 indifferent between $p_1 \in [p_L, p_H]$ it must be satisfied that

$$\Pi_1(p_1) = p_1[\frac{p_2(\tilde{x}(p_1)) - p_1}{t}] = \overline{\Pi}. \qquad (2)$$

By solving (2) for $\tilde{x}(p_1)$ and using (1) we obtain the pricing policy for the discriminatory firm $p_2(x) = \frac{\overline{\Pi}}{x} + tx$. Figure 1 shows the equilibrium policies when $\overline{\Pi} = \frac{t}{8}$. The lowest price that allows firm 1 to obtain the profit $\overline{\Pi}$ is $p_L = \overline{\Pi}$ and, at this price, firm 1 would capture the whole market. The price p_H depends on the consumer reservation value. Denote by x_H firm 1´s market share such that $\frac{\overline{\Pi}}{x_H} + tx_H =$ R, that is x_H is the quantity sold by firm 1 when it charges a price p_H. It is easy to check that x_H and p_H, as a function of $\overline{\Pi}$, are given by:

$$x_H(\overline{\Pi}) = \frac{R - \sqrt{R^2 - 4t\overline{\Pi}}}{2t} \qquad (3)$$

$$p_H(\overline{\Pi}) = \frac{R + \sqrt{R^2 - 4t\overline{\Pi}}}{2} \tag{4}$$

Note that given the discriminatory firm´s pricing policy $p_2(x) = \frac{\overline{\Pi}}{x} + tx$, the mill pricing firm is indifferent between prices $[p_L, p_H]$. It is easy to check that the discriminatory firm maximizes profits when the mill firm charges the highest price p_H.[12] Therefore the maximum profit that the discriminatory firm can obtain, maintaining a profit $\overline{\Pi}$ for firm 1, is given by:

$$\Pi_2(\overline{\Pi}) = \int_{\frac{R - \sqrt{R^2 - 4t\overline{\Pi}}}{2t}}^{1} \{ \frac{\overline{\Pi}}{x} + tx - t(1 - x)\} dx$$

This profit can be expressed as

$$\Pi_2(\overline{\Pi}) = -\overline{\Pi}\ln x_H(\overline{\Pi}) + tx_H(\overline{\Pi})[1 - x_H(\overline{\Pi})]$$

Given that the pricing policy of the leader must satisfy $p_2(x) \geq t\,(1 - x)$, since it has to cover its transportation costs, firm 1 might guarantee a profit of $t/8$ by charging a price $p_1 = t/2$. Therefore, fim 2´s pricing policy, that is $\overline{\Pi}$, solves the following problem:

$$\max_{\overline{\Pi}} \; -\overline{\Pi}\ln x_H(\overline{\Pi}) + tx_H(\overline{\Pi})[1 - x_H(\overline{\Pi})] \tag{5}$$
$$\text{s.t.} \quad \overline{\Pi} \geq \frac{t}{8}$$

It is straightforward to show that the objective function is concave and that $\frac{\partial \Pi_2(t/8)}{\partial \overline{\Pi}} > 0$. The first order condition is given by:

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[12] Due to the optimal pricing policy of the discriminatory firm holds the mill pricing firm indifferent between prices $[p_L, p_H]$, so in order for firm 1 to choose p_H its profits at prices $[p_L, p_H)$ must be ε-below $\overline{\Pi}$. We have to change the discriminatory pricing policy slightly: $p_2(x) = \{ \frac{\overline{\Pi}}{x} + tx - \delta$, for $x_H < x \leq 1$; R for $x \leq x_H \}$, with $\delta > 0$, $\delta \to 0$.

$$\Pi_2'(\overline{\Pi}) = -\ln x_H(\overline{\Pi}) - \overline{\Pi}\frac{x_H'(\overline{\Pi})}{x_H(\overline{\Pi})} + t x_H'(\overline{\Pi})[1 - 2x_H(\overline{\Pi})] = 0 \qquad (6)$$

Given (3), condition (6) can be rewritten as:

$$-\ln\left[\frac{R - \sqrt{R^2 - 4t\overline{\Pi}^*}}{2t}\right] - \frac{2t\overline{\Pi}^*}{\sqrt{R^2 - 4t\overline{\Pi}^*}\left[R - \sqrt{R^2 - 4t\overline{\Pi}^*}\right]}$$

$$+\frac{t}{\sqrt{R^2 - 4t\overline{\Pi}^*}}\left[1 - \frac{\left[R - \sqrt{R^2 - 4t\overline{\Pi}^*}\right]}{t}\right] = 0 \qquad (7)$$

Therefore, the backward induction solution is given by:

$$p_2^*(x) = \frac{\overline{\Pi}^*(R)}{x} + tx$$

$p_1^* = p_H(\overline{\Pi}^*(R)) = \dfrac{R + \sqrt{R^2 - 4t\overline{\Pi}^*(R)}}{2}$, and the equilibrium profits are

$$\Pi_1^* = \Pi_1^{UD} = \overline{\Pi}^*(R)$$

$$\Pi_2^* = \Pi_2^{UD} = \int_{\frac{R - \sqrt{R^2 - 4t\overline{\Pi}^*(R)}}{2t}}^{1} \{\frac{\overline{\Pi}^*(R)}{x} + tx - t(1 - x)\}dx \qquad (8)$$

Condition (7) defines the optimal $\overline{\Pi}^*(R)$ as an implicit function of the reservation value. It is easy to check that $\overline{\Pi}^*$ is an increasing function of *R*. However, it is not possible to obtain an explicit expression for $\overline{\Pi}^*$ from condition (7). For this reason we consider a numerical approximation. With no loss of generality we normalize *t* to be *t* = 1 and evaluate the firms´ profits at different levels of the reservation value. Table 2 summarizes the equilibrium profits as a function of *R*, so we can approximate the value of $\overline{\Pi}^*$ that maximizes the discriminatory firm´s profit for each *R*. We obtain the following equations of linear regression (where ρ^2 is the determination coefficient) :

$$\overline{\overline{\Pi}}^{*} = -0.15064 + 0.36548R \qquad \rho^2 = 0.9998 \qquad (9)$$

$$\Pi_2^{*} = 0.098288 + 0.36777R \qquad \rho^2 \cong 1 \qquad (10)$$

Note that (9) and (10) provide a very good aproximation of equilibrium profits. When firm 2 is the mill pricing firm and firm 1 the discriminatory firm we obtain symmetric results, thus, $\Pi_1^{UD} = \Pi_2^{DU}$ and $\Pi_1^{DU} = \Pi_2^{UD}$. Given these profits, Table 1 summarizes the possible outcomes of the second stage and the following proposition states the main result of this subsection.

Proposition 1.- If the discriminatory firm is the leader when firms choose different pricing policies, the equilibrium analysis of the pricing policy game depends on the consumers reservation value, *R*. (i) When $t < R \leq \overline{R}$ the pricing policy game has two Nash equilibria in pure strategies: either both firms price uniformly or both firms price discriminate. (ii) When $\overline{R} < R < \overline{\overline{R}}$ spatial price discrimination is a dominant strategy and the pricing policy game is a Prisoner´s Dilemma. (iii) When the consumer reservation value is large enough, $R > \overline{\overline{R}}$, the pricing policy game has two Nash asymmetric equilibria in which one firm prices according to f.o.b. and the other price discriminates. The critical levels of the reservation value are given by: $\overline{R} = 1.0923t$ and $\overline{\overline{R}} = 1.0962t$.

Proof. Given (9), (10) and Table 1, we have three possibilities:

(i) When $t < R \leq \overline{R}$ the equilibrium profits are such that $\Pi_1^{UU} > \Pi_1^{DU} > \Pi_1^{DD} > \Pi_1^{UD}$ and $\Pi_2^{UU} > \Pi_2^{UD} > \Pi_2^{DD} > \Pi_2^{DU}$. Therefore, there are two Nash equilibria: (U, U) and (D, D).

(ii) When $\overline{R} < R < \overline{\overline{R}}$ the equilibrium profits are such that $\Pi_1^{DU} > \Pi_1^{UU} > \Pi_1^{DD} > \Pi_1^{UD}$ and $\Pi_2^{UD} > \Pi_2^{UU} > \Pi_2^{DD} > \Pi_2^{DU}$. Therefore, (D, D)* is the unique Nash equilibrium and price discrimination is a dominant strategy.

(i) When $R \geq \overline{\overline{R}}$, the equilibrium profits are such that $\Pi_1^{DU} > \Pi_1^{UU}$, $\Pi_1^{UD} > \Pi_1^{DD}$ and $\Pi_2^{UD} > \Pi_2^{UU}$, $\Pi_2^{DU} > \Pi_2^{DD}$. Thus, there are two Nash equilibria: (U, D) and (D, U). Q.E.D.

Note that when the discriminatory firm is the leader, if $t < R \leq \overline{R}$ the pricing policy game is not a Prisoner´s Dilemma and, in fact, there are two equilibria in pure strategies (and another in mixed strategies). If one firm is committed to f.o.b. the best response for the other firm is f.o.b. as well. If one firm has chosen to be flexible and produce all the varieties of the product, the best response for the other firm is delivered pricing as well. When $\overline{R} < R < \overline{\overline{R}}$ spatial price discrimination is a dominant strategy and the pricing policy game is a Prisoner´s Dilemma. When $R > \overline{\overline{R}}$, the pricing policy game has two Nash asymmetric equilibria in which one firm prices according to f.o.b. and the other price discriminates. This case is the most relevant given the critical levels of the reservation value, $\overline{R} = 1.0923t$ and $\overline{\overline{R}} = 1.0962t$. Figure 2 graphically summarizes the above proposition.

Therefore, the conclusion of Thisse and Vives (1988) that there is a robust tendency for a firm to choose the discriminatory policy does not hold when the discriminatory firm is the price-leader .

5. **Simultaneous price competition**

The above game provides us with an intuition as to the equilibrium outcome when mixed strategies are allowed in the simultaneous pricing game. We obtained in the previous subsection that the equilibrium pricing policy of the discriminatory firm is such that the mill pricing firm is indifferent between prices belonging to the interval $[p_L(\overline{\Pi}), p_H(\overline{\Pi})]$. In order to find an equilibrium in the simultaneous game, we would only need to prove that there exists a distribution function for the mill pricing firm with support in an interval $[p_l(\tilde{\Pi}), p_h(\tilde{\Pi})]$ such that the best response of the discriminatory firm to that mixed strategy is $p_2(x, \tilde{\Pi}) = \frac{\tilde{\Pi}}{x} + tx$. Lemma 2 and 3 give us some properties that the equilibrium must satisfy.

Lemma 2. In the market area of the mill pricing firm the full price of the mill firm at $p_h(\tilde{\Pi})$ is lower or equal to the transportation cost from the discriminatory firm. That is, $p_h(\tilde{\Pi}) + tx \leq t(1 - x)$ for $x \in [0, x_h]$, where x_h denotes the marginal consumer at p_h.

Proof. If this condition is not satisfied the discriminatory firm might undercut the full price of the mill pricing firm in order to capture a greater market area.

Lemma 3. The value of $\tilde{\Pi}$ is $\frac{t}{8}$.

Proof. The value of $\tilde{\Pi}$ cannot be less than $\frac{t}{8}$ since firm 1 (the mill pricing firm) can always ensure this profit. Note that if it charges a price $p_1 = \frac{t}{2}$ then its market area will never be less than $\frac{1}{4}$ given that the discriminatory firm must cover transportation costs. Therefore $\tilde{\Pi} \geq \frac{t}{8}$. On the other hand, Lemma 2 implies that the function $p_2(x, \tilde{\Pi}) = \frac{\tilde{\Pi}}{x} + tx$ cannot be always above $t(1 - x)$ (however, the intersection between $p_1 + tx$ and

$p_2(x)$ would be over $t(1-x)$ and Lemma 2 would not be satisfied). The values of $\tilde{\Pi}$ that satisfy this condition are $\tilde{\Pi} \leq \frac{t}{8}$. Thus we can conclude that $\tilde{\Pi} = \frac{t}{8}$.

Lemma 4. The support of the mixed strategy for the mill pricing firm is the interval $[\frac{t}{8}, \frac{t}{2}]$.

Proof. Note that the lower extreme of the support must satisfy $p_l \geq \frac{t}{8}$ given that $p_1 = \frac{t}{8}$ is the highest price that allows firm 1 to capture the whole market and to obtain a profit $\tilde{\Pi} = \frac{t}{8}$. From Lemma 2 we know that the intersection between $p_1 + tx$ and $p_2(x)$ cannot be over $t(1-x)$. In order for this condition to be satisfied it is neccessary that $p_h \leq \frac{t}{2}$. Finally if we do not consider the complete interval the discriminatory firm could always change its strategy in order to obtain more profits.

Lemma 5. The following strategies constitute a mixed Nash equilibrium with an associated profit $\tilde{\Pi} = \frac{t}{8}$ for the mill pricing firm:

(i) The cumulative distribution function for the mill firm $F_1^*(p_1) = 1 - k\frac{e^{-\frac{t}{t-2p_1}}}{t-2p_1}$ where $k = \frac{3}{4}te^{\frac{4}{3}}$ with support $[\frac{t}{8}, \frac{t}{2}]$.

(ii) The delivered pricing policy

$$p_2^*(x) = \begin{cases} t(1-x) & \text{for } x \in [0, \frac{1}{4}) \\ \frac{t}{8x} + tx & \text{for } x \in [\frac{1}{4}, 1] \end{cases} \text{ for the discriminatory firm.}$$

Proof. See Appendix.

The density function is given by

$$f_1^*(p_1) = \frac{dF_1^*(p_1)}{dp_1} = \frac{3p_1 t}{(t-2p_1)^3} e^{(-\frac{t}{t-2p_1}+\frac{4}{3})}$$

The following proposition states the main result of this section.

Proposition 2. Under simultaneous price competition in all the subgames, the pricing policy game has two Nash equilibria in pure strategies: either both firms price uniformly (f.o.b.) or both firms price discriminate.

Proof. The expected profit of the discriminatory firm is given by

$$\Pi_2^{UD} = \Pi_2^e(t/8) = \int_{\frac{t}{8}}^{\frac{t}{2}} \left\{ \int_{\frac{t}{8p_1}}^{1} [\frac{t}{8x} + tx - t(1-x)]dx \right\} dF_1^*(p_1)$$

where $F_1^*(p_1)$ is given by Lemma 5. Assume that the mill firm follows a pure strategy $p_1 = \frac{t}{2}$ (maintaining the discriminatory firm pricing policy $p_2^*(x) = \frac{t}{8x} + tx$) then the secure profit for firm 2 would be

$$\Pi_2 = \int_{\frac{1}{8}}^{\frac{1}{2}} \frac{t}{8x} + tx - t(1-x)]dx = \frac{3+2\ln 4}{16} t$$

Therefore

$$\Pi_2^{UD} = \Pi_2^e(t/8) < \frac{3+2\ln 4}{16} t < \frac{t}{2} = \Pi_2^{UU}$$

and from Table 1 we conclude that the pricing policy game has two Nash equilibria in pure strategies: both firms price uniformly or both firms price discriminate. Q.E.D.

6.- **Concluding remarks**

We have shown that the general tendency for firms to price discriminate found by Thisse and Vives (1988) crucially depends on the rules of price competition. In particular, spatial price discrimination is a dominant strategy only when the mill pricing firm is the leader and the discriminatory firm the follower. When the leader-follower roles are reversed, equilibrium pricing policies depend on the consumer´s reservation value. Under simultaneous price competition in all subgames, we find a mixed strategies equilibrium when firms choose different pricing policies and we demonstrate that the pricing policy game has two perfect Nash equilibria: both firms price uniformly or both firms price discriminate.

Appendix

Proof of Lemma 5

We seek to obtain a distribution function for the mill pricing firm with support $[\frac{t}{8},\frac{t}{2}]$ such that the best response of the discriminatory firm to that mixed strategy is $p_2^*(x)=\begin{cases} t(1-x) & \text{for } x\in[0,\frac{1}{4}) \\ \frac{t}{8x}+tx & \text{for } x\in[\frac{1}{4},1] \end{cases}$. Given that the discriminatory firm can charge a different price at each point of the market, we solve the profit maximization problem at a generic point.

The discriminatory firm sells the product to the consumer located at x if its delivered price, $p_2(x)$, is lower than or equal to the full price of the mill pricing firm, p_1+tx. Therefore the probability of this event is $P(p_1+tx\geq p_2(x))=P(p_1\geq p_2(x)-tx)$ = $1-F_1(p_2(x)-tx)$, where $F_1(p_2(x)-tx)$is the distribution function of the mill pricing firm evaluated at $p_2(x)-tx$. So the expected profit of the discriminatory firm at x is given by $\Pi_2^e(x)=[p_2(x)-t(1-x)][1-F_1(p_2(x)-tx)]$. The first order condition of the maximization problem is

$$\left.\frac{\partial\Pi_2^e(x)}{\partial p_2(x)}\right|_x=[1-F_1(p_2(x)-tx)]-[p_2(x)-t(1-x)]f_1(p_2(x)-tx)=0 \qquad \text{(A1)}$$

where $f_1(p_2(x)-tx)$ is the density function. (A1) can be rewritten as

$$[1-F_1(p_2(x)-tx)]=[p_2(x)-t(1-x)]f_1(p_2(x)-tx) \qquad \text{(A2)}$$

We want to obtain the density function $f_1(.)$ such that $p_2{}^*(x)=\frac{t}{8x}+tx$ is a solution for this maximization problem. By substituting this value in (A2) we get

$$[1-F_1(\frac{t}{8x})]=(\frac{t}{8x}+2tx-t)f_1(\frac{t}{8x}) \tag{A3}$$

If we denote $\frac{t}{8x}=z$ (A3) can be expressed as

$$[1-F_1(z)]=(\frac{z^2+\frac{t^2}{4}-tz}{z})f_1(z) \tag{A4}$$

Given that $f_1(z)=F_1'(z)$, then (A4) is a variable coefficient first order linear differential equation. It is straightfoward to check from the solution of this differential equation that the equilibrium distribution function for the mill firm is given by

$F_1^*(p_1)=1-k\frac{e^{-\frac{t}{t-2p_1}}}{t-2p_1}$ where $k=\frac{3}{4}te^{\frac{4}{3}}$

and the density function is $f_1^*(p_1)=\frac{dF_1^*(p_1)}{dp_1}=\frac{3p_1t}{(t-2p_1)^3}e^{(-\frac{t}{t-2p_1}+\frac{4}{3})}$. Q.E.D.

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		Firm 2: U	Firm 2: D
Firm 1	U	$\frac{t}{2}, \frac{t}{2}$	Π_1^{UD}, Π_2^{UD}
	D	Π_1^{DU}, Π_2^{DU}	$\frac{t}{4}, \frac{t}{4}$

Table 1. Summary of firms' profits.

Reservation Value	$\overline{\Pi}^*$	Π_2^*
1	0.225	0.46648
2	0.58	0.83379
3	0.94	1.20142
4	1.31	1.56917
5	1.67	1.93695
6	2.04	2.30479
7	2.41	2.67263
8	2.77	3.04047
9	3.14	3.40833
10	3.51	3.77619

Table 2.- Equilibrium profits.

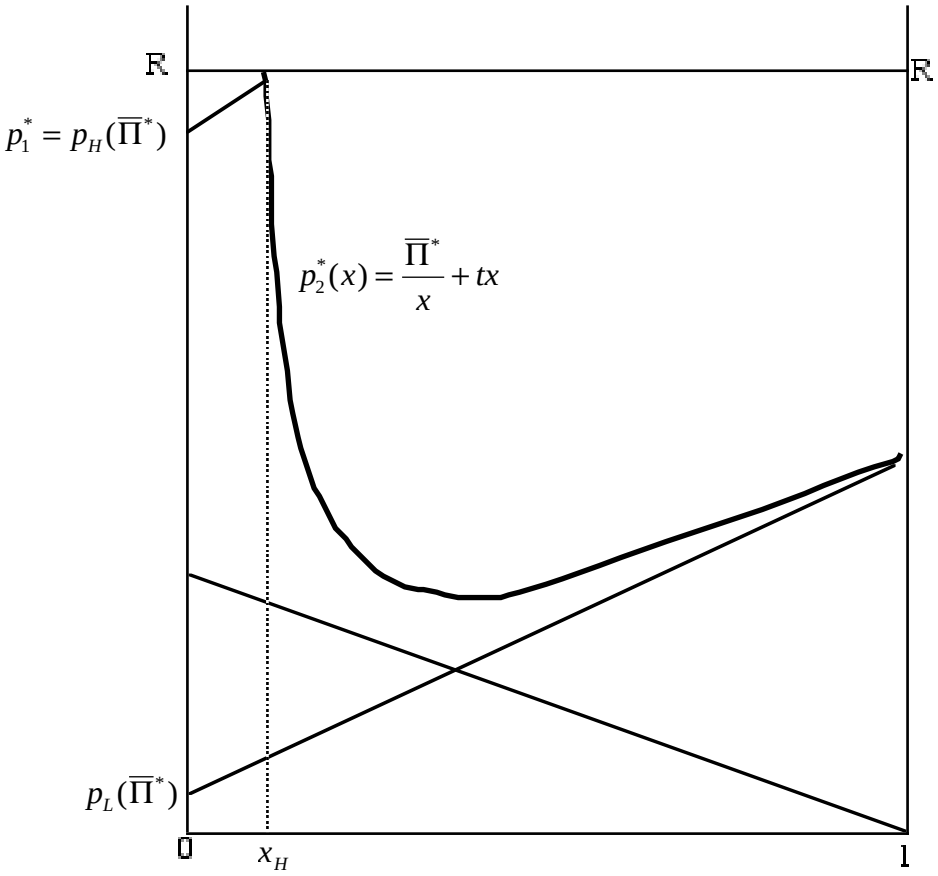

Figure 1. Equilibrium pricing policies when the mill pricing firm is the follower.

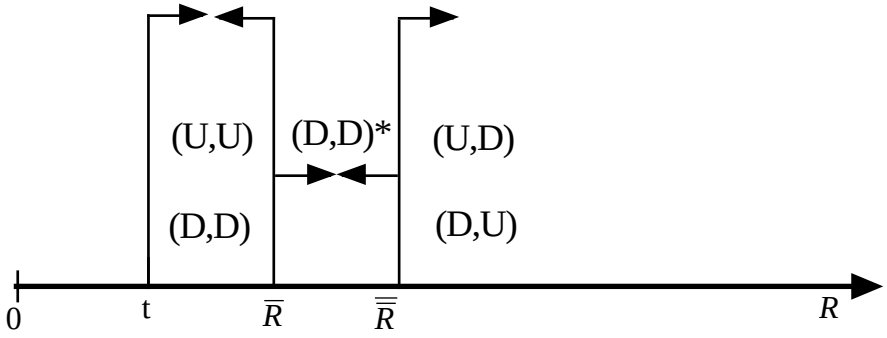

Figure 2. Equilibrium pricing policies as a function of the reservation value. The asterisk denotes Nash equilibrium with dominant strategies (Prisoners´ Dilemma).