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Analytical analysis of the magnetic field, heat generation and absorption, viscous dissipation on couple stress casson hybrid nano fluid over a nonlinear stretching surface

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ABSTRACT

The aim of this research paper is to study two-dimension flow of Casson hybrid nanofluid **along** with magnetic field, heat generation and absorption, and viscous dissipation on a nonlinear extending surface. The primary goal of this study is to improve the heat transfer relationship, which is in high demand in the manufacturing and engineering industries. The outputs of this study will be used to reduce the energy consumption in industry and other engineering fields,for example, the achievement of energy is not enough, but also to adjust the consumptions of energy and this is possible only to approve the development heat transmission liquids to mechanism the expenditures of energy and to improvement. The described similarity transformation is used to convert the non-dimensionless form of the nonlinear partial differential equation to the dimensionless form of the nonlinear ordinary differential equation. An approximate analytical method is used to solve the derived dimensionless form of nonlinear ordinary differential equations, one for velocity and the other for temperature. Graphs are used to highlight the most relevant results acquired from velocity and temperature. Tables are used to describe the skin friction coefficient and the Nusselt number.

1. Introduction

Real fluids are fluids for which viscosity is non-zero, examples of the real fluids are all the natural fluid are Real fluids. Newtonian fluids satisfy Newton's law of viscosity, whereas non-Newtonian fluids satisfy the power-law model. Toothpaste, blood, ketchup, Lucite paint, drilling mud's, and biological fluids are examples of non-Newtonian fluids. In industries sector and manufacturing developments magneto hydrodynamic (MHD) flow has significant application. The researchers take more interest in non-Newtonian fluids as compared to Newtonian fluids due to some important uses. The applications to these fluids and the analysis of hybrid nanofluid flow are helpful in many areas like engineering, technology and industries etc. It has showed many outcomes and hybrid nanofluids flow exertion has been realized in many areas such as shifting from explicit situations in the flow in lungs of humans to lubricant nuisance in industries. This maybe one of the principal subfield of hybrid nanofluids flow troubles, Study of thin fluid flow is practical application. It is an attractive interface between structural mechanics, theology and fluid mechanics. Link cable and fibber primer is one of its noteworthy applications. The fine uses of these applications are metals exclusion, streaked of victuals, steady forming, drawing of elastic sheets, and chemical equipment's used in treatment. The perception to these uses, the objective has been the analysis of the fluid films on the extending surfaces. Using peristaltic flow, Mekheimer et al. [1] investigate induced magnetic field heat and mass transport. Ellahi and Riaz [2] talk about how they used third-grade MHD flow analysis.

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Mukhopadhyay and Gorla [3] use an extended surface to explore magneto hydrodynamic boundary layer flow. Nadeem et al. [4] used spreading surface to study three dimensional non-Newtonian Casson fluids. Benazir et al. [5] they study time dependent magneto hydrodynamics Casson fluid by using vertical cone. Parmar et al. [6] they study non-Newtonian Casson fluid model for blood. Nadeem et al. [7] they discuss the influence of chemical reaction and radiation for non-Newtonian Casson fluid. Ullah et al. [8] they study non-Newtonian Casson fluid over a permeable surface. Kamran et al. [9] they investigated on-Newtonian. Archana et al. [10] they study non-Newtonian Nano fluid along with the influence of radiation. Gireesha et al. [11] they study non-Newtonian Casson fluid along with the influence of chemical reaction. Souayeh et al. [12], Ullah et al. [13], Aziz and Afify [14]. Hybrid Nano fluid is defined as combination of two or more nanoparticle suspended in the base fluid. Suresh et al. [15] they give the idea of for the first time that hybrid Nano fluids more progressive features as compared to ordinary Nano fluid. Devi and Devi [16,17] they study hybrid Nano fluid by using extending surface to boost up heat transfer. Tayebi and Chamkha [18] they study $Cu - Al_2O_3 -$ water hybrid Nano fluid by using an annulus. Ghadikolaei et al. [19] They discuss $TiO_2 - Cu/H_2O$ hybrid Nano fluid along with resistance force. Hayat et al. [20] they study $Ag - CuO/$water hybrid Nano fluid by using revolving disc. Yousefi et al. [21], they study aqueous titanic-copper hybrid by using extending cylinder. Subhani and Nadeem [22] they study $Cu - TiO_2/H_2O$ hybrid Nano fluid by using extending sheet. Hayat et al. [23] Thy study the influence of cross diffusion on MHD flow. Takhar et al. [24] they study MHD flow along with mass diffusion and chemical reaction over an extending surface. Awais and Hayat [25] they study unsteady Casson fluid by using extending surface. Vyas and Ranjan [26] hey investigate s MHD flow by using non-linear stretching sheet over permeable medium. Sandhya, G et al. [27] they used exponentially stretching sheet to study MHD flow. Khan, J et al. [28] they study numerical study of nanofluid flow and heat transfer over a rotating disk using Buongiorno's model. Siddiqui, a et al. [29] they study a new theoretical approach of wall transpiration in the cavity flow of the ferro fluids. Turkyilmazoglu, M. et al. [30] they study the transparent effects of Buongiorno nanofluid model on heat and mass transfer. Wahid, N. S et al. [31] they study MHD hybrid Cu–Al2O3/water nanofluid flow with thermal radiation and partial slip past a permeable stretching surface: analytical solution. Khan, M et al. [32] they study the numerical simulation of stagnation point flow of non-Newtonian fluid (Carreau fluid) with Cattaneo-Christov heat flux. Khan, M et al. [33] they study numerical solution of MHD flow of power law fluid subject to convective boundary conditions and entropy generation. Hayat, T et al. [34] they study Theoretical investigation of chemically reactive flow of water-based carbon nanotubes (single-walled and multiple walled) with melting heat transfer. Khan, M et al. [35] they study transportation of CNTs based nanomaterial flow confined between two coaxially rotating disks with entropy generation. Khan, M et al. [36] they study Transportation of heat through Cattaneo-Christov heat flux model in non-Newtonian fluid subject to internal resistance of particles. Ijaz Khan et al. [37] they study Binary chemical reaction with activation energy in rotating flow subject to nonlinear heat flux and heat source/sink. Chu, Y et al. [38] they study Thermophoresis particle deposition analysis for nonlinear thermally developed flow of Magneto-Walter's B nanofluid with buoyancy forces. Nazeer, M et al. [39] they study Mathematical modeling of bio-magnetic fluid bounded within ciliated walls of wavy channel. Khan, a et al. [40] they study Non-Newtonian based micro polar fluid flow over nonlinear starching cylinder. Lanjwani, H et al. [41] they study Stability analysis of triple solutions of Casson nanofluid past on a vertical exponentially stretching/shrinking sheet. Abbas, N et al. [42] they study Heat and Mass Transfer of Micro polar-Casson Nanofluid over Vertical Variable Stretching Riga Sheet. Abbas, N et al. [43] they study Theoretical study of non-Newtonian micro polar nanofluid flow over an exponentially stretching surface with free stream velocity. Abbas, N et al. [44] they study Numerical analysis of unsteady magnetized micro polar fluid flow over a curved surface. One of the most series issue now a day, and most of the researchers trying to study the advance resources to reestablish the energy and to regulator the feeding of heat transfer devices. All These resources are depending with solids, gasses and liquids. To increase the thermal efficiency of the common liquids is possible through the addition of the small solid metal particles. There are many liquids with low heat transfer rate which have some useful used in different engineering these liquids are bio-liquid, polymeric solution, various oils, greases, water, toluene, refrigerants, ethylene glycol and so on. The important aim of this study is to deliver the idea of boost of the heat transfer which is used in the new technology. In our study the authors used hybrid nanofluid for the improvement of heat transfer ratio which paly important role. For example, the achievement of energy is not enough, but also to adjust the consumptions of energy and this is possible only to approve the development heat transmission liquids to mechanism the expenditures of energy and to improvement. The most heat transmission which is the demand of the industry and other related scientific fields. The flow problem analysis on the bases of converting nonlinear partial differential equations to nonlinear ordinary differential equation by using similarity transformation. The model nonlinear differential equations have been analyses by approximate analytical method. The impact of different developing results has been inspected with help of figure and tables. The novelty of the present research article is for the first time the Impact of Casson hybrid nanofluid **along** with magnetic field, heat generation, absorption, viscous dissipation with couple stress flow for this model on a nonlinear extending surface is study analytically, the comparison of the present research work is presented in the form of table with the already publish work, the Mathematica software is used to obtained the basic result, the convergence control parameter is discussing in table form for both velocity and temperature equations.

2. Mathematical formulation

Consider 2D steady laminar incompressible flow of $TiO_2 + Ag +$ Casson and $TiO_2 + Ag$ hybrid Nano fluid across a permeable extending surface, the extending surface of the sheet is takin along *x axis* and *y axis* is perpendicular to the surface. The fluid flow occur across a permeable extending surface when $y \geq 0$, which activated when the surface is extending. It is assumed that the flow of the fluid particles at any point on the surface is direct proportional to the power of the distance from the split, $u_w(x) = ax^n$ represent velocity of the stretching surface where n represent stretching parameter, a is constant and x represent the distance along the stretching sheet T_w, represent the surface temperature, T_∞, represent ambient temperature where $T_w > T_\infty$. The governing equation for continuity, velocity and temperature and all the assumption are settled [45].

2.1. Subject to the boundary condition

$$\begin{aligned} &u = u_w + u_s, v = \pm v_w , T = T_\infty \; at \; y = 0 \\ &u \to 0, \; T \to T_\infty \; at \; y \to \infty \end{aligned} \tag{4}$$

where α_m shows thermal conductivity, u *and* v shows the velocity along x and y direction, ρ shows the density of the fluid, σ, shows electrical conductivity, B_0, shows magnetic induction, v_w shows the suction and injection velocity and Q_0 shows heat generation coefficient.

To transform the non dimensionless from of the differential equation to dimensionless form we used the flowing similarity transformation.

$$\eta = y\frac{1}{\beta}\sqrt{\frac{a(n+1)}{2\nu}}x^{\frac{n-1}{2}}, u = ax^n f'(\eta)$$
$$\nu = -\sqrt{\frac{\nu a(n+1)}{2}}x^{\frac{n-1}{2}}\left[f(\eta) + \left(\frac{n-1}{n+1}\right)\eta f'(\eta)\right] \tag{5}$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

Putting Equ 5 in Equ1,2 and 3, Equ5 fulfilledEqu1 and convertEqu2 and 3 to the subsequent form

$$\left(1+\frac{1}{\beta}\right)\frac{(1-\varphi_1-\varphi_2)^{-2.5}}{(1-\varphi_1-\varphi_2)+\varphi_1\left(\frac{\rho_1}{\rho_{hnf}}\right)+\varphi_2\left(\frac{\rho_2}{\rho_{hnf}}\right)}f''' + ff'' - \left[\frac{2n}{n+1}\right]f'^2 - (1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}Mf' - Kf^{v} = 0 \tag{6}$$

$$Pr^{-1}\frac{(1-\varphi_1-\varphi_2)^{-2.5}}{(1-\varphi_1-\varphi_2)+\varphi_1\left(\frac{\rho_1}{\rho_{hnf}}\right)+\varphi_2\left(\frac{\rho_2}{\rho_{hnf}}\right)}\theta'' + f\theta' + (1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}Nt\theta'^2 + EcQ\theta f''^2 = 0 \tag{7}$$

The transform boundary conditions are

$$f = F_w, f' = 1, f'' = 1+\xi_p, \theta = 1 \ at\ \eta = 0$$
$$f' \to 0, \theta \to 0\ ,\ at\ \eta \to \infty \tag{8}$$

β shows Casson parameter $\Pr = \frac{\nu_{hnf}}{\alpha_m}$, shows prandtl number $Nt = \frac{(\rho c_p)_{hnf} D_B (T_w - T_\infty)}{(\rho c_p)_{hnf}\nu_{hnf}T_\infty}$, shows thermophoresis parameter, $F_w = \frac{-\nu_w}{\sqrt{\frac{a\nu(n+1)}{2}}x^{\frac{n-1}{2}}}$, shows suction injection parameter, $\xi_p = l\sqrt{\frac{a(n+1)}{2}}x^{\frac{n-1}{2}}$, shows slip parameter for liquids, $M = \frac{\sigma_{hnf}B_0^2}{\rho_{hnf}}\frac{2}{\alpha(n+1)x^{n-1}}$, shows magnetic field parameter $Q = \frac{2Q_0}{\rho c_p \alpha(n+1)x^{n-1}}$, shows heat generation/absorption coefficient, $\mathrm{Ec} = \frac{U_m^2}{c_p(T_m - T_0)}$. Represent Eckert number, and K,represent Couple stress parameter,

The nondimensionless form of skin frication and Nusselt number are

$$C_f = \frac{\mu_{hnf}\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\rho_{hnf}U^2}, Nu = \frac{1}{k_f(T_w - T_\infty)}\left(k_{hnf}\frac{\partial T}{\partial y}\right)_{y=0}, \tag{9}$$

Using equation (5) in equation (9) we have the dimensionless form of skin frication and Nusselt number

$$C_f(\mathrm{Re}_x\mathrm{Pr}) = \left(\frac{(1-\varphi_1)^{2.5}.(1-\varphi_2)^{-2.5}}{(1-\varphi_2)\left\{\left((1-\varphi_1)+\varphi_1.\left(\frac{\rho_{s_1}}{\rho_f}\right)+\varphi_2.\left(\frac{\rho_{s_2}}{\rho_f}\right)\right)\right\}}\right)f''(0), \tag{10}$$

$$Nu = -\frac{k_{hnf}}{k_f}\theta'(0), \tag{11}$$

2.2. 2a. Solution by HAM

The Homotopy Analysis Method (HAM) is among semi-analytical methods which has many applications for solving various problems of engineering and basic science. For the first time Liao discussed this method for solving mathematical problems [46–48]. Also several applications of the HAM can be found in []

In order to solve Eqs. (6) and (7) along with the boundary conditions (8), we apply the HAM with the following technique. The solutions having the auxiliary parameters $\hbar$, which adjust and control the convergence of the solutions.

The initial guesses are selected as follows

$$f_0(\eta) = \eta\ and\ \ \theta_0(\eta) = 1 \tag{12}$$

The linear operators are taken as L_f and L_θ:

$$L_f(f) = f'''\ and\ \ L_\theta(\theta) = \theta'' \tag{13}$$

which have the following properties:

$$L_f\left(c_1 + c_2\eta + c_3\eta^2\right) = 0\ and\ \ L_\theta(c_4 + c_5\eta) = 0 \tag{14}$$

where $c_i(i = 1-7)$ are the constants in general solution:

The resultant non-linear operatives N_f *and* N_θ are given as:

$$N_f[f(\eta;p)] = \left(1+\frac{1}{\beta}\right)\frac{\partial^3 f(\eta;p)}{\partial\eta^3} + f(\eta;p)\frac{\partial^2 f(\eta;p)}{\partial\eta^2} - \left[\frac{2n}{n+1}\right]\left(\frac{\partial f(\eta;p)}{\partial\eta}\right)^2 - M\frac{\partial f(\eta;p)}{\partial\eta} - K\frac{\partial^5 f(\eta;p)}{\partial\eta^5} = 0 \tag{15}$$

$$N_\theta[f(\eta;p),\theta(\eta;p),\varphi(\eta;p)] = \frac{1}{\Pr}\frac{\partial^2\theta(\eta;p)}{\partial\eta^2} - f(\eta;p)\frac{\partial\theta(\eta;p)}{\partial\eta} + Nt\left(\frac{\partial\theta(\eta;p)}{\partial\eta}\right)^2 + EcQ\theta(\eta;p)\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2 = 0 \tag{16}$$

The basic idea of HAM is described in Refs. [46–49], the zero$^{\text{th}}$ -order problems from Eqs. (6) and (7) are:

$$(1-p)L_f[f(\eta;p) - f_0(\eta)] = p\hbar_f N_f[f(\eta;p)] \tag{17}$$

$$(1-p)L_\theta[\theta(\eta;p) - \theta_0(\eta)] = p\hbar_\theta N_\theta[f(\eta;p),\theta(\eta;p),\varphi(\eta;p)] \tag{18}$$

The equivalent boundary conditions are:

$$f(\eta;p)|_{\eta=0} = 0,\ \left.\frac{\partial f(\eta;p)}{\partial\eta}\right|_{\eta=0} = 1,\ \left.\frac{\partial^2 f(\eta;p)}{\partial\eta^2}\right|_{\eta=\beta} = 0,$$
$$\theta(\eta;p)|_{\eta=0} = 1,\ \left.\frac{\partial\theta(\eta;p)}{\partial\eta}\right|_{\eta=\beta} = 0, \tag{19}$$

where $p \in [0,1]$ is the imbedding parameter, $\hbar_f$ and $\hbar_\theta$ are used to control the convergence of the solution. When $p = 0$ and $p = 1$ we have:

$$f(\eta;1) = f(\eta)\ and\ \ \theta(\eta;1) = \theta(\eta) \tag{20}$$

Expanding $f(\eta;p)$ *and* $\theta(\eta;p)$ in Taylor's series about $p = 0$

$$f(\eta;p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m, \tag{21}$$

$$\theta(\eta;p) = \theta_0(\eta) + \sum_{m=1}^{\infty}\theta_m(\eta)p^m, \tag{22}$$

where

$$f_m(\eta) = \frac{1}{m!}\left.\frac{\partial f(\eta;p)}{\partial\eta}\right|_{p=0}\ \text{and}\ \ \theta_m(\eta) = \frac{1}{m!}\left.\frac{\partial\theta(\eta;p)}{\partial\eta}\right|_{p=0} \tag{23}$$

The secondary constraints $\hbar_f$ and $\hbar_\theta$ are chosen in such a way that series (22) converges at $p = 1$, switching $p = 1$ in (22), we obtain:

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta),$$
$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty}\theta_m(\eta), \tag{24}$$

The m th-order problem satisfies the following:

$$L_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_f R_m^f(\eta),$$
$$L_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_\theta R_m^\theta(\eta), \tag{25}$$

The corresponding boundary conditions are:

$$f_m(0) = F_w\ , f_m''(0)\ = 1 + \xi_p, \theta_m(0) = 1\ at\ \eta = 0$$
$$f_m'(0) \to 0, \theta_m'(0) \to 0\ at\ \eta \to \infty \tag{26}$$

$$R_m^f(\eta) = \left(1 + \frac{1}{\beta}\right) f_{m-1}''' + \sum_{k=0}^{m-1} f f_{m-1-k}'' - \left[\frac{2n}{n+1}\right] \left(f_{m-1-k}'\right)^2 - M \sum_{k=0}^{m-1} f_{m-1-k}' K \times \sum_{k=0}^{m-1} f_{m-1-k}^{v} = 0 \tag{27}$$

$$R_m^\theta(\eta) = \frac{1}{\Pr}\theta_{m-1}'' + \sum_{k=0}^{m-1} f\theta_{m-1-k}' + Nt \sum_{k=0}^{m-1} \left(\theta_{m-1-k}'\right)^2 + EcQ \sum_{k=0}^{m-1} \theta\left(f_{m-1-k}''\right)^2 = 0 \tag{28}$$

3. Result and discussion

The important aim of this study is to deliver the idea of boost of the heat transfer which is used in the new technology. In our study we will used different type of hybrid nanofluied for the improvement of heat transfer ratio which paly important role. For example, the achievement of energy is not enough, but also to adjust the consumptions of energy and this is possible only to approve the development heat transmission liquids to mechanism the expenditures of energy and to improvement. The most heat transmission which is the demand of the industry and other related scientific fields. The second and important feature of this research is the enhancement of heat to reduce energy consumptions. The flow problem is conducted using shrinking and stretching surface. The outputs of this study will be used to reduce the energy consumption in industry and other engineering fields. The purpose of the current research article is to use the hybrid Nano fluids for the boost up of heat transfer ratio. In our research paper we used new type of Nano fluid known as (hybrid Nano fluid), in our combination TiO_2+ Ag+ Casson represent hybrid Nano fluidand TiO_2 + Ag represent Nano fluid. The solid elements dissolve in the base liquid and after its constant diffusion the hybrid Nano fluid is produced. The total results of the given flow problem are presented through Figs. [2–8,50], the influence of different parameters on velocity profile is presented in figs [2–5], the influence of different parameters on temperature profile is presented in figures [6–8, 50], Fig. 1 shows geometry of the defined flow problem, Table 1,Table 2 signify the convergence control parameter for both velocity and temperature for the defined problem,while Table 3, Table 4 represent the effect of different parameter on Skin friction and Nusselt number respectively.

Table 5 shows the comparison of the present work with already publish work for velocity equation.

Table 6 HAM and numerical comparison for $\theta(\eta)$.

Fig. 2 shows variation in M (magnetic field parameter) on velocity distribution for both TiO_2 + Ag + Casson and TiO_2 + Ag , from Fig. 2, we observed that velocity is declining function of M or there is inverse relation between magnetic field parameter and velocity distribution that is growing magnitude of magnetic field parameter decrees the velocity distribution. Physically by increasing magnetic field parameter the resistive types of forces known as Lorentz forces are produced. The strength of such forces enhances with the rising strength of magnetic field parameter M. Which counteracts the motion of fluid within boundary layer and drop the thickness of boundary layer. Also these forces created resistance to the flow of fluid particle moments and as a result the velocity filed is decreasing as shows in Fig. 2, this type of flow is used for hotness purpose in industries sector.

Fig. 3 shows the variation in Couple stress K on velocity profile for both TiO_2 + Ag + Casson and TiO_2 + Ag , we noted from figure, 4,that velocity is the decreasing function of Couple stress K or there is inverse relation between Couple stress K and velocity distribution that is the increasing magnitude of Couple stress K decrees the velocity distribution. Physically by increasing Couple stress K produced viscous forces, these forces have direct relation to viscosity, so by increasing these forces produced resistance to flow of fluids particles, so as a result the velocity of the fluids particles is decreasing as shows in Fig. 4, this type of flow is used for hotness purpose in industries sector.

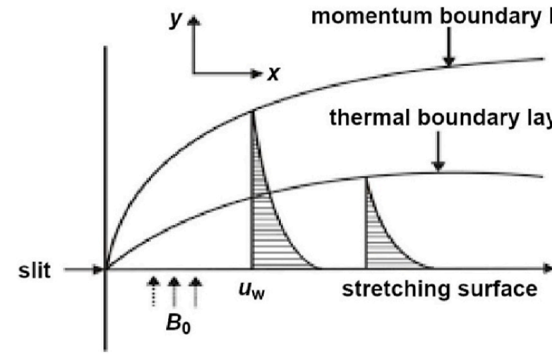

Fig. 1. Geometry of the flow problem.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu_{hnf}\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_{hnf}}u - \frac{\nu_{hnf}}{\rho_{hnf}}\frac{\partial^4 u}{\partial y^4}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \nabla^2 T + \frac{1}{\rho c_p}Q_0(T - T_\infty) + \frac{\mu_{hnf}}{(\rho cp)_{hnf}}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Table 1
Shows the convergence of the defined problem for velocity equation, from table two we see that as we increase the number of iteration the residual error is decreasing and strong convergence is obtained.

m	$f'(\eta)$
5	0.0411×10^{-1}
10	0.0914×10^{-2}
15	0.0713×10^{-4}
20	0.0818×10^{-5}
25	0.0987×10^{-6}

Table 2
Shows the convergence of the defined problem for temperature equation, from table three we see that as we increase the number of iteration the residual error is decreasing and strong convergence is obtained.

m	$\theta(\eta)$
5	0.1691×10^{-1}
10	0.0126×10^{-2}
15	0.0218×10^{-3}
20	0.0956×10^{-4}
25	0.1983×10^{-5}

Table 3
Consequenceof different parameters such as M, K, n, ξ on Skin frication, from table four we see that Skin frication is the decreasing function of M, K, n, ξ. Actually by increasing these parameters the surface frication forces are increasing and as a result Skin friction is increasing as shown in table four.

M	K	n	ξ	C_f
0	0.01	0.1	0.1	0.0357
1				0.0513
2				0.0791
	0.02			0.0801
	0.03			0.0830
		0.5		0.0876
		0.9		0.0894
			0.4	0.0952
			0.7	0.0991

Table 4
Consequenceof $\Pr, N_t, Ec, \xi$ on Nusselt number, from table five we see that Nu is the growing function of $\Pr, N_t, Ec, \xi$. Actually by enhancing $\Pr, N_t, Ec, \xi$ heat energy is stored in hybrid Nano fluid because of the drug or frictional force and hence Nusselt number is enhances.

Pr	Ec	N_t	ξ	Nu
0	1	3	0.1	0.0185
			0.5	0.0254
			0.9	0.0452
3				0.0532
5				0.0612
	1.5			0.0731
	2			0.0793
		5		0.0842
		7		0.0921

Table 5
HAM and numerical comparison for $f(\eta)$.

m	Numerical	OHAM	Absolute Error
1	1.0000	1.0000	0.0000
2	0.2372	0.2045	0.0327
3	0.2791	0.2611	0.0180
4	0.3632	0.3373	0.0259
5	0.6721	0.6529	0.0192
6	0.8381	0.8012	0.0369
7	0.3937	0.3851	0.0086
8	0.5737	0.5567	0.0170
9	0.9429	0.9012	0.0417
10	0.7331	0.7073	0.0258

Table 6
Shows the comparison of the present work with already publish work for temperature equation.

m	Numerical	OHAM	Absolute Error
1	1.0000	1.0000	0.000
2	0.2847	0.2732	0.0155
3	0.2887	0.2457	0.0430
4	0.2371	0.1024	0.1347
5	0.4179	0.4011	0.0168
6	0.4400	0.4375	0.0025
7	0.5567	0.5329	0.0238
8	0.5933	0.5719	0.0214
9	0.6453	0.6267	0.0186
10	0.7547	0.7397	0.0150

Fig. 4 shows variation in β (Casson parameter) on velocity distribution for both $TiO_2 + Ag + $ Casson and $TiO_2 + Ag$, from Fig. 2, we observed that velocity is declining function of β (Casson parameter) or there is inverse relation between magnetic field parameter and velocity distribution that is growing magnitude of magnetic field parameter

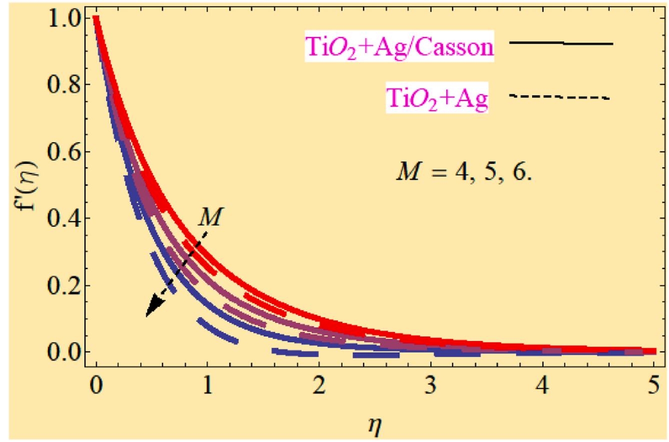

Fig. 2. Influence of magnetic field parameter on velocity profile.

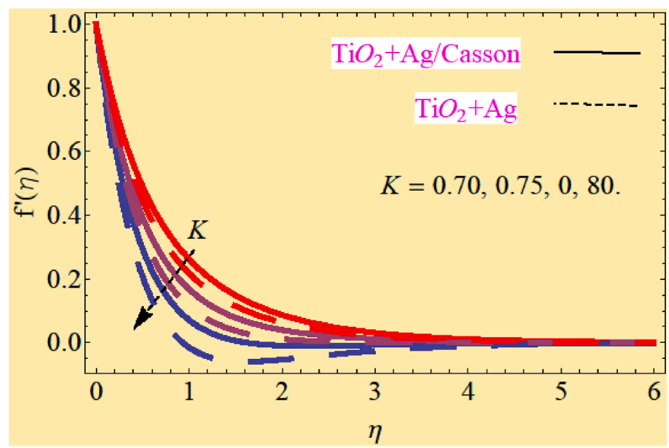

Fig. 3. Influence of Couple stress parameter on velocity profile.

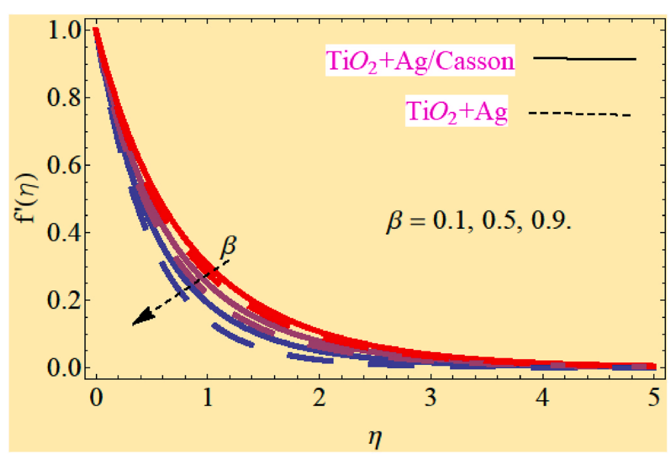

Fig. 4. Influence of Casson parameter on velocity profile.

decrees the velocity distribution. Physically by increasing β (Casson parameter) the viscous forces are produced. The strength of such forces enhances with the rising strength of β (Casson parameter). Which counteracts the motion of fluid within boundary layer and drop the thickness of boundary layer. Also these forces created resistance to the flow of fluid particle moments and as a result the velocity filed is decreasing as shows in Fig. 4, this type of flow is used for hotness purpose in industries sector.

Fig. 5 shows the impact of nonlinear stretching parameter on velocity distribution for both $TiO_2 + Ag +$ Casson and $TiO_2 + Ag$, from Fig. 5 we noted that the relation between nonlinear stretching parameter and velocity distribution is direct relation or velocity is the increasing function of nonlinear stretching parameter. Physically nonlinear stretching parameter have inverse relation to the viscosity, that is by increasing solar radiation parameter decrees the viscosity of fluid and as a result the fluid particles move easily and velocity profile is increasing or here, the thermal boundary layer converges faster than the momentum boundary layer as shown in Fig. 5.

Fig. 6 shows the relation between Eckert number and temperature profile for both $TiO_2 + Ag +$ Casson **and** $TiO_2 + Ag$, from Fig. 6 we see that temperature profile is the increasing function of Eckert number that is as we increase Eckert number temperature field is increasing as shows in Fig. 8,physically by increasing the Eckert number. It will enhance the kinetic energy due to this intermolecular collision is increasing, so the temperature profile is increasing, Eckert number can be used as a hot agent.

Fig. 7 shows the influence slip parameter and temperature filed for both $TiO_2 + Ag +$ Casson and $TiO_2 + Ag$, Figure indicate that temperature filed is the rising function slip parameter. This impact is due to improving the slip parameterthe fluid particles move quickly and as a result temperature filed is increasing.

Fig. 8 shows the relation between thermophoresis parameter and temperature profile for both $TiO_2 + Ag +$ Casson and $TiO_2 +$ Ag , from Fig. 8 we noted that temperature profile is the growing function of thermophoresis parameter,that is by increasing thermophoresis parameter temperature filed is increasing. Energy coming to structure element will increase its temperature until heat losses are able to balance heat gain. Heat losses occur at the expense of emission, convective transport to the atmosphere and transmission to adjacent areas, caused by heat-conduction. The higher the temperature of the heated element in comparison to surrounding objects, the more significant the heat losses,therefore, enhancing the relaxation parameter heat energy is stored in the system, and hence $\theta(\eta)$ enhances.

Fig. 9 that temperature is the decreasing function Pr as shown in Fig. 9, physically thickness of the momentum boundary layer to be larger than that of the thermal boundary layer, or that the viscous diffusion is larger than the thermal diffusion and therefore, the larger amount of the Prandtl number reduces the thermal boundary layer and so by increasing Pr decreases in temperature filed, prandtl number can be used as a cooling agent.

4. Conclusion

Analytical investigation of magnetic field, heat generation and absorption, and viscous dissipation on Couple stress Casson hybrid Nano fluid across a nonlinear stretching surface, this type of flow has some important application in industries sector and engineering sector for the purpose of cooling and hotness effect, also hotness and cooling play some important role in daily life. The similarity transformation was used to convert the non-dimensional form of differential equation to the dimensionless form of differential equation. The approximation analytical method is used to solve the differential equations in their dimensionless form. In future we will face the problem to adjust the

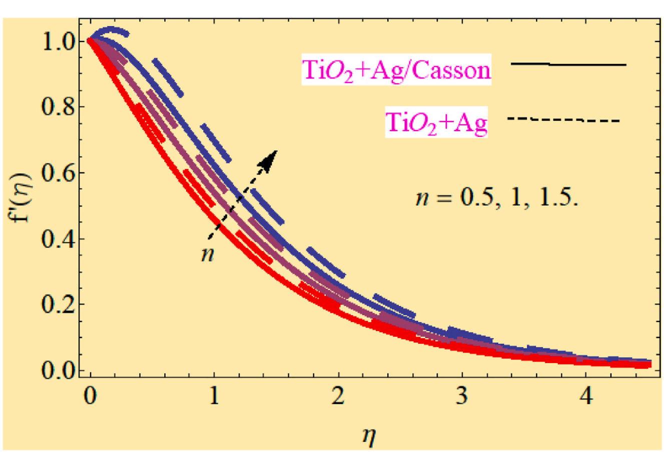

Fig. 5. Influence of nonlinear stretching parameter on velocity profile.

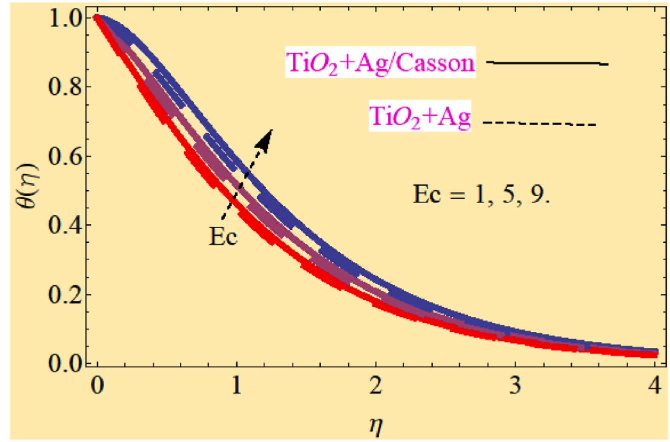

Fig. 6. Influence Eckert number on temperature profile.

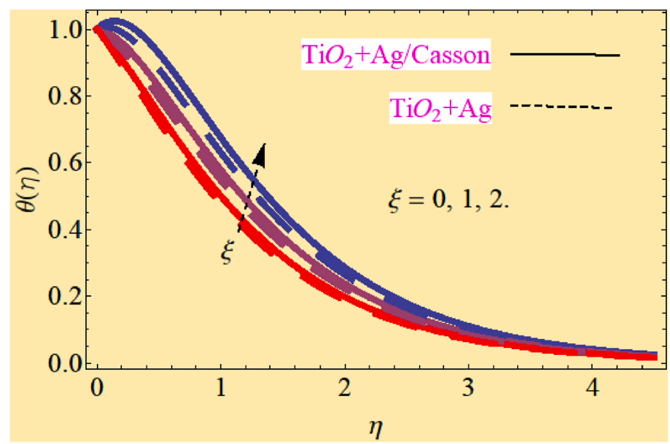

Fig. 7. Influence slip parameter on temperature profile.

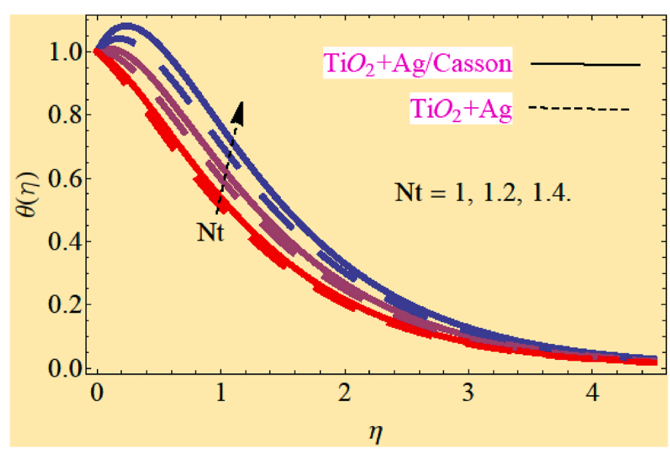

Fig. 8. Influence thermophoresis parameter on temperature profile.

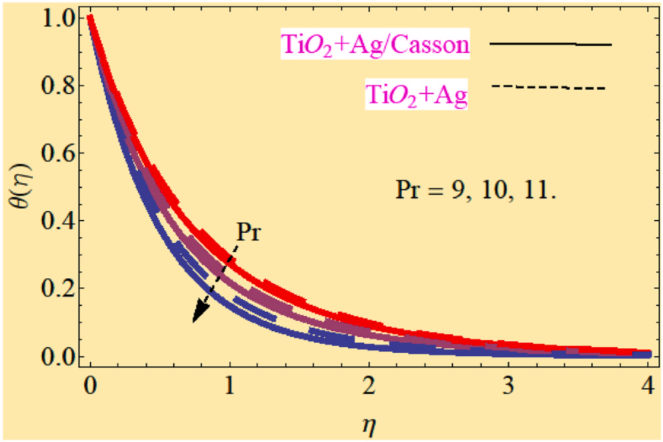

Fig. 9. Influence Prandtl number on temperature profile(θη).

consumptions of energy, in our research article we used hybrid nanofluid which helped in the enhancement of heat transfer ratio which have some important applications, for example, the achievement of energy is not enough, but also to adjust the consumptions of energy and this is possible only to approve the development heat transmission liquids to mechanism the expenditures of energy and to improvement. The most heat transmission which is the demand of the industry and other related scientific fields. The following are the main findings of this study:

1. In the velocity field, the value of the stretching parameter is increasing.
2. As the value of the magnetic field parameter rises, the velocity filed decreases.
3. As the Casson parameter rises, the velocity filed decreases.
4. In velocity filed, the value of Couple stress parameter declarations is increasing.
5. The temperature declarations' Prandtl number declarations are increasing in value.
6. Thermophoresis parameter value increases as temperature rises.
7. Temperature rises as the Eckert number rises.
8. Temperature rises as the value of the slip parameter rises.

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Declaration of competing interest

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