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Flight delays in Germany: a model for evaluation of future cost risk

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Air traffic has been increasing in Germany over the last decades reaching in 2018 an all-time high with more than 3 million flights. This increase has led to a rise in delays, which generate different costs to airlines, passengers, and Air Navigation Service Providers. This paper focuses on understanding and predicting these costs. For this purpose, a stochastic modelling method is proposed to estimate future air traffic, delays and the cost of future delays. The model allows to better understand what the full distribution of the delay costs may look like. To that end, the paper analyses 1,826 daily items (from 1/1/2014 to 12/31/2018) with information of air traffic and delays for German airspace. Findings suggest that overall mean delay costs for 2019 may be up to 280 million €, while in the 5% worst cases this value could go up to an average of 319 million €.

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1. Introduction

The airspace sector has a great influence on today's economy and society. The enormous increase in air traffic, technical and commercial innovations and the development of costly infrastructures are indicators of the strength of the sector. The European airspace is one of the most vibrant sectors in the world with more than thirty thousand flights a day and a very high density of airports. However, there are also significantly increased complexities related to its management, such as turnaround operations at airports (Wu & Caves, 2000, 2007; Herrera & Moreno, 2011), flight schedule punctuality (Wu & Caves, 2010; Eurocontrol, 2019a), airport apron capacity (Mirkovic & Tosic, 2013; Jiménez et al., 2013), operational and organizational performance management (Choong, 2013; Bourne et al., 2014: Bezerra & Gomes, 2016) and baggage handling systems (Cavada et al., 2017).

Air traffic management takes place through a relatively complex pattern with a wide variety of activities and actors who are involved within an overarching system (see SESAR (2018)). Airports are, together with aircraft operators and users, one of these crucial elements that needs to be orchestrated in order to ensure the proper functioning of the whole system. Air navigation service providers (ANSP)[1] have great interest that the whole value chain works seamless with adequate airport management and predictable airspace user intentions. These are, therefore, key issues that require to be carefully examined if we are to determine the optimal and necessary resources needed to deal with an increasing current and future demand. Moreover, it is crucial to better understand the changes that the whole system is undergoing in order to better adapt to the coming future needs. In all this complexity, delays[2] are both a consequence of many interacting factors and the reason for many other disruptions in the system.

Earlier studies on the air sector have focused, for instance, on aircraft noise effects (Janssen & Vos, 2011; Basner et al., 2017; Chatelain &Van Vyve, 2018; Ganic et al., 2018; Ho-Huu et al., 2019; Otčenášek, 2019), on the links between air traffic emissions and health (Yim et al., 2015; Harrison et al., 2015; Vujovic & Todorovic, 2017), and on taxi fuel consumption (Nikoleris et al., 2011; Khadilkar & Balakrishnan, 2012; Vaishnav, 2014). Other pieces of work include the impact of fragmentation (Ansuategi et al., 2019), air traffic volatility on efficiency (Standfuss et al., 2021), climate costs due to air management (Goicoechea et al., 2021) or the assessment of airline's competitive position at the network level (Maertens, 2020). More directly linked to cost of delays several pieces of work have tried to assess these (Cook, 2004; Cook & Tanner, 2011, 2015; Koster et al., 2014; Nikoleris & Hansen, 2015; Performance Review Unit, 2019). These earlier efforts have quantified the cost values for several ranges of delays in minutes under a number of different parameters and conditions but have not tried to assess or forecast total cost of the delays in Europe.

This paper is the first attempt to estimate the total costs of delays for German air space taking into account real data on the distribution of delays during 2014 to 2018[3]. For this purpose, a stochastic model is proposed to explain how delays are distributed, that is, what the probability of each delay range occurring is. By including information on delay costs per minute this paper allows us to shed some light on the distribution of the cost of delays and later to forecast how these may look in the future. This is a much needed contribution to the literature.

The paper analyses daily air traffic and delay information for Germany in period 2014-2018. These values are used to simulate 20,000 air daily traffic movements for the year 2019, the delays and the associated costs. The estimation are compared with 2019 actual data. The proposed model includes

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[1] These are public or private legal entities providing Air Navigation Services, i.e. managing air traffic on behalf of a company, a region or country. These services usually include, among others: (1) Air Traffic Management, (2) Communication navigation and surveillance systems or (3) Meteorological service for air navigation.

[2] 334 million passengers (+26%) were impacted by ATFM delays and cancellations, which resulted in a total cost for the EU economy of €17.6 bn. (+28%), in comparison with 2017 (Eurocontrol, 2019b).

[3] The German ANSP provider is the Deutsche Flugsicherung (DFS). More information in https://dfs.de/.

three main components: (1) a stochastic model of the number of flights, composed of a stochastic daily part of delays and a deterministic daily part that includes seasonality (annual and semi-annual), trend, weekend and constant components; (2) a Tobit model that relates the number of flights to the expected delays; and (3) a cost model that allows simulating the full distribution of economic losses of delays using a stochastic approach. Other stochastic models have been proposed earlier in other areas such as marine transport (Abadie et al., 2017; Abadie and Goicoechea, 2019a), but as far as we are aware, the proposed model in this paper is a novelty in the air transport sector, and it is also the first time that the stochastic modelling (to simulate future daily flights) is combined with a Tobit model (to estimate delays)[4].

The rest of the paper is organised as follows: Section 2 presents the data used in the study. Section 3 presents the details of the modelling efforts and results. Section 4 concludes and suggests further research.

2. The data

The information available consists in 1,826 daily items with information on air traffic and delays in Germany (see the evolution in Figure 1) used for model calibration. This information covers the period beginning the 1st of January of 2014 and ending the 31st of December of 2018, that is, 5 years. An additional set of 365 days corresponding to 2019 is used for testing the model. The data was collected by Deutsche Flugsicherung (DFS).

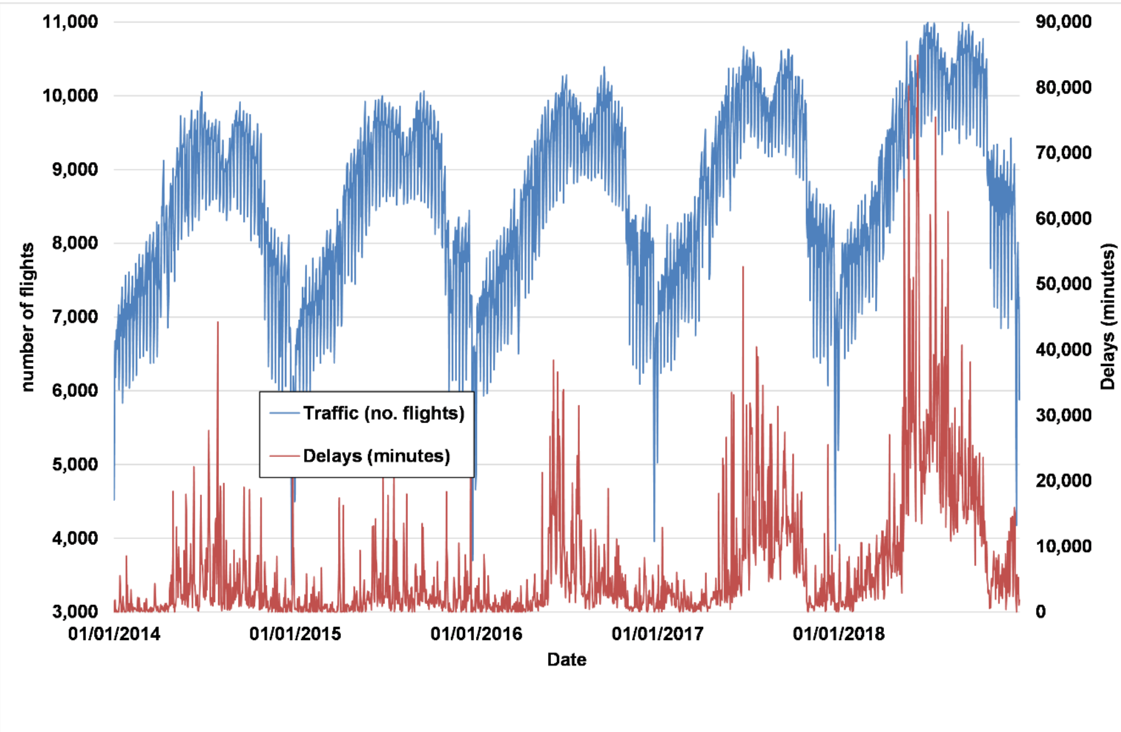

Figure 1. Number of daily flights and delays Germany (from 1/1/2014 to 12/31/2018)

[4] Other studies have recently examined and applied different quantitative methods and simulation approaches to forecast traffic demand and assist in transport infrastructure investment decisions under scenarios of risk and uncertainty (Jin et al., 2020; Solvoll et al., 2020; Oliveira et al., 2020; Barreto Martins & Strambi, 2021; Polat & Battal, 2021; Tascón & Díaz Olariaga, 2021). However, none of them apply a stochastic model for determining future cost risk of delays. Stochastic models take uncertainty into account while providing both distributions of the target variables as well as its expected values. These distributions allow us obtaining risk measures such as percentiles and/or mean values for the worst cases. The calculations of stochastic models is far more complex but the precision as well as the quality of the results provided is much better (see Wilmott, 2000; Hull, 2014).

Figure 1 shows a positive correlation between number of flights and delays, the latter defined as the number of minutes between the expected and actual time of arrival. Figure 2 is a scatter plot of these two variables showing that a high number of flights usually leads to longer delays.

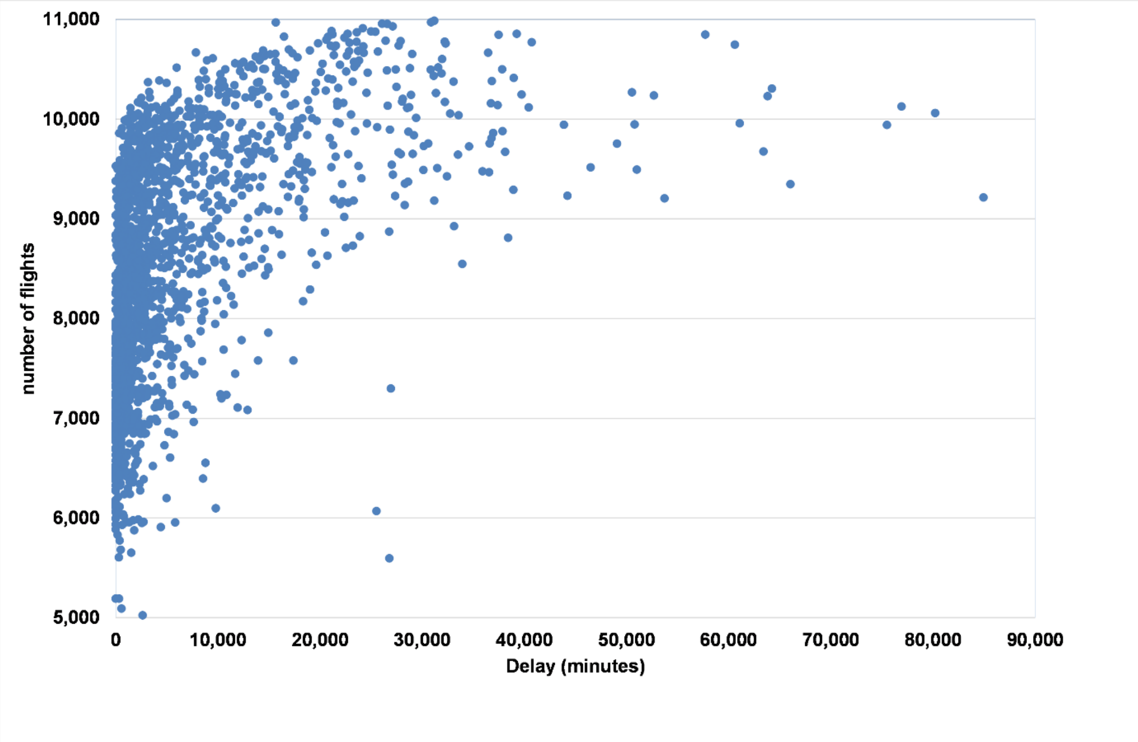

Figure 2. Scatter plot Number of Daily Flights and delays Germany (from 1/1/2014 to 12/31/2018)

The Pearson correlation between flights and delays has been calculated to be 0.491. Note, however, that some days there were a significant number of delays with only few delays in other days. This indicates that the behaviour of this relationship is a bit more complex and requires further analysis. Table 1 below shows some summary statistic for the data. There are a minimum of 3,370 flights and a maximum of 8,712 flights. The standard deviation is a measure of volatility showing that for delays is significantly higher than for air traffic data (or number of flights). The skewness is a measure of the asymmetry of the probability distribution. The traffic negative skewness indicates that its left tail is longer that the right tail. The positive skewness of delays shows that the right tails of its distribution is longer that its left tail. The positive values of excess kurtosis in the delay variable informs that the tails on this distribution is heavier than that of a normal distribution, this distribution shape generate higher impacts of extreme events. Also the summary statics shown the variable percentiles, the 90% of traffic values are between 6,467 and 10,456.

Table 1. Summary statistics

	Mean	Median	Minimum	Maximum
Traffic (No)	8,614.8	8,712.0	3,370.0	11,024
Delay (mins)	6,545.5	2,607.0	0.0	84,953
	Standard deviation	Coefficient of variation	Skewness	Excess Kurtosis
Traffic (No)	1,249.5	0.14504	-0.4646	-0.0509
Delay (mins)	9,760.2	1.4911	3.0001	12.589
	5% percentile	95% percentile	IQ range	Missing observations
Traffic (No)	6,467.4	10,456	1,873.5	0
Delay (mins)	0.0	26,825	7,328.0	0

3. Modelling efforts and calibration

As mentioned earlier, the proposed model has three main components worth explaining in detail: 1) a stochastic model for the number of flights, 2) a Tobit model to relate the number of flights to the expected delays, and 3) a cost model to estimate the economic losses due to delays. The combination of the three components allows us to estimate and depict the full distribution of the cost of delays. Figure 3 below illustrates the calculation process that is described in the rest of the section. Note that this is a very valuable information as one can then analyse the probability of different economic cost ranges occurring as a consequence of air traffic delays. This information should be of great interest for air traffic planners, because not only the expected values are calculated, also their distributions as well as their associated risk measures. This very important contribution as it allows to undertake risk-based planning such as the case of planning based on reacting reasonably to 95% of the cases. This kind risk management of approach is rather usual in many areas such as financial, energy or climate risk economics.

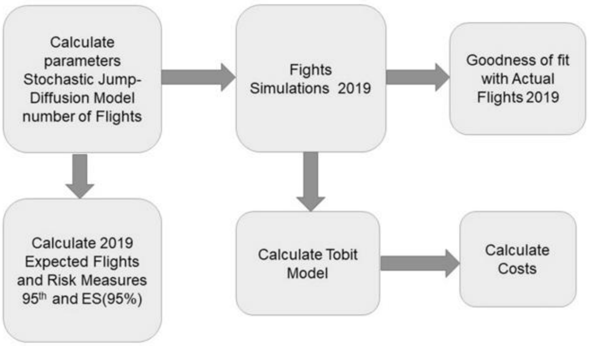

Figure 3. Calculation process scheme

3.1 Stochastic model of air traffic

In order to model the air traffic with precision, this paper proposes combining both a stochastic component and a deterministic part into the modelling efforts. This is done by noting the natural logarithm of the number of flights, n_t, in Equation (1) as the sum of the two components: the first one, $f(t)$, is the deterministic that contains both the seasonality and the trend, while the second one, X_t, is the mean reverting jump diffusion stochastic model. The time elapsed t from the initial moment to a specific day is measured in years.

$$ln(n_t) = f(t) + X_t \quad (1)$$

Deterministic part

$$f(t) = \beta_1 \sin(2\pi t) + \beta_2 \cos(2\pi t) + \beta_3 \sin(4\pi t) + \beta_4 \cos(4\pi t) + \beta_5 t + \beta_6 D_t + \beta_7 \quad (2)$$

where Equation (2) describes the deterministic part of Equation (1), and includes annual and semi-annual seasonal components of air traffic, a trend, a constant and a dummy variable D_t for weekends (Fridays, Saturdays and Sundays), where $D_t = 1$ if it is a weekend and $D_t = 0$ otherwise. Note that using an annual and semi-annual sine and cosine components we get a better fit of the deterministic part than using only annual sines and cosine. Calibrating the seven first parameters with daily data of air traffic we obtain the results presented in Table 2.

Table 2. Deterministic parameters calculated with data on daily flights

Parameter	Value	95% confidence interval
β_1	-0.0532	-0.0584 – -0.0480
β_2	-0.1576	-0.1627 – -0.1524
β_3	-0.0054	-0.0105 – -0.0002
β_4	-0.0402	-0.0453 – -0.0350
β_5	0.0285	0.0259 – 0.0310
β_6	-0.0623	-0.0697 – -0.0550
β_7	9.0053	8.9973 – 9.0133

Some parameters can be further used to undertake future sensitivity analysis, as in the case of the trend β_5. Figure 4 below shows the natural logarithm of number of flights together with the estimated deterministic part of the model. One can note that the model fits very well with the data. That is, the deterministic part explains a very significant part of the behaviour of flights. The regression R-squared value is 0.7407 and the adjusted R-squared is 0.7399.

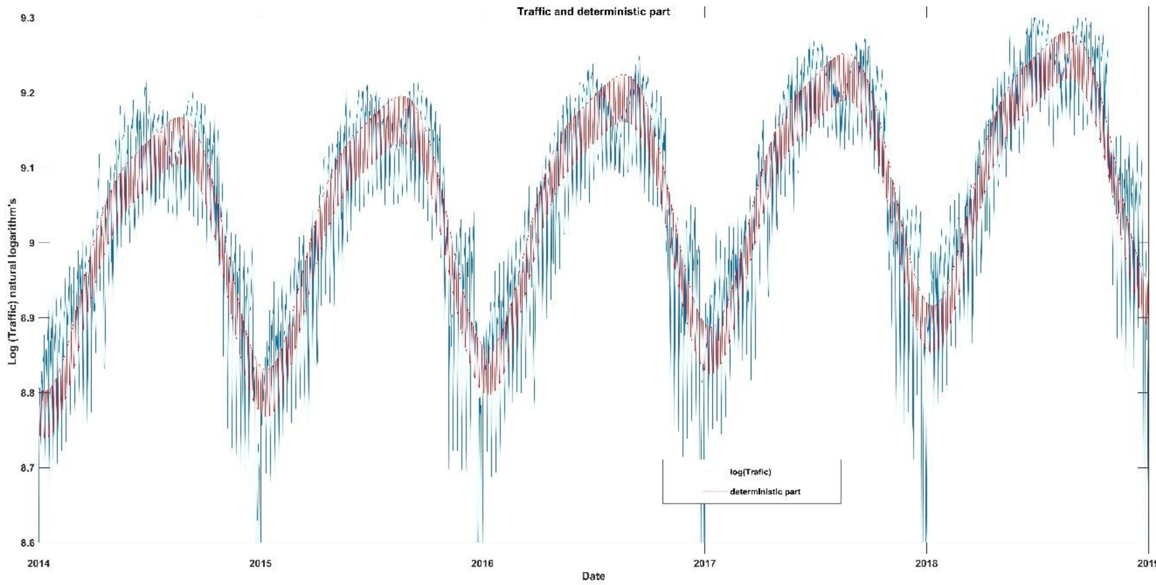

Figure 4. Natural logarithm of number of flights and the estimated deterministic part

In attempt to better illustrate this in Figure 4, we now show in Figure 5 a much closer look to the data for the year 2018. The Figure clearly shows that the deterministic part of the model (that is seasonality, trend, and weekend days) allows us to predict a significant part of the number of flights in any given date. In a closer look at deterministic part in Figure 4, red line, one can note that impact of weekend days as air traffic reaches maximum values.

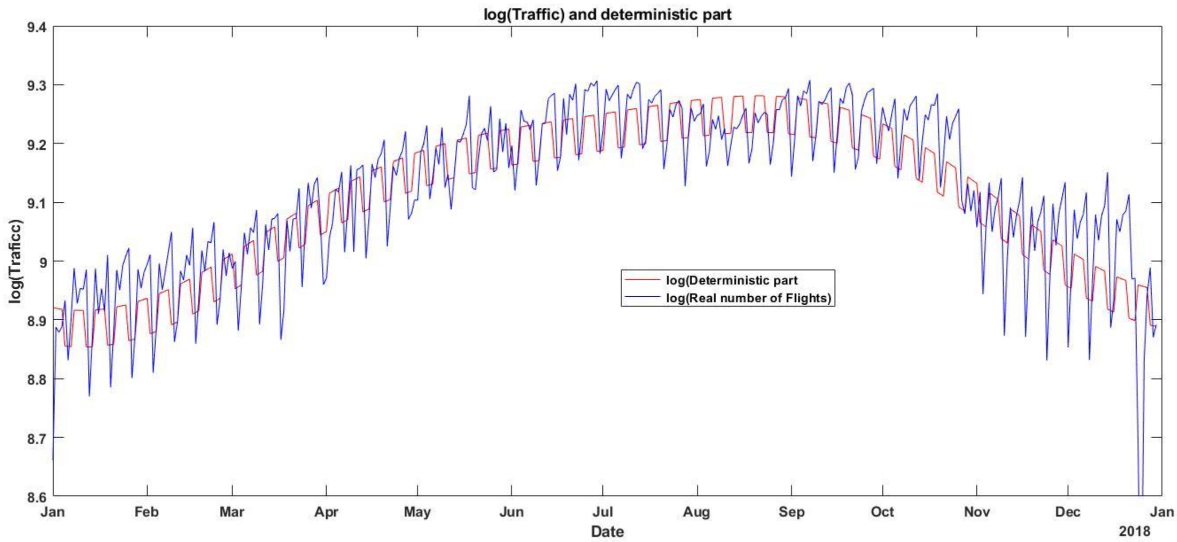

Figure 5. Number of flights (2018) and its deterministic part

However, it is also clear that air traffic has a strong stochastic component that should be considered.

Stochastic part

If we now move to the stochastic part of the model as illustrated in equation (1), we can plot the logarithm of the stochastic part of number of flights in Figure 6 by subtracting to the natural logarithm the determinist part modelled earlier. In this case one can note the negative jumps that show important reduction in the number of flights in certain dates that are to be considered in any estimation effort.

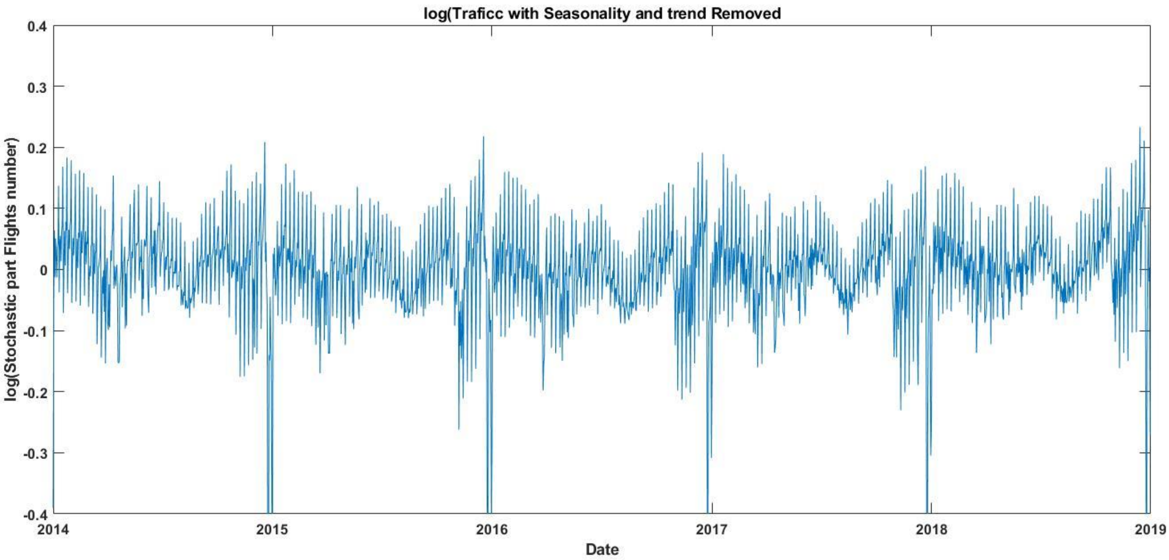

Figure 6. Stochastic part of flight traffic from 2014 to 2019

By looking at Figure 6 it can be noted that the values fluctuate close to the mean value which is zero. In addition, when values deviate from the mean value, they tend to revert towards equilibrium (mean) values. This behaviour strongly justifies the use of a mean-reverting model as suggested here. An additional feature to note are the important jumps towards negative values. Both features together firmly recommend the use of the well-known Ornstein-Uhlenbeck mean reverting model with jumps (see Abadie & Chamorro, 2013, for more detail on this). This model is described in Equation (3) below.

$$dX_t = (\alpha - \kappa X_t)dt + \sigma dW_t + J(\mu_j, \sigma_j)dq_t \qquad (3)$$

In Equation (3) the current logarithm of number of flights (stochastic part) tends to level α/κ in the long term, with a speed of reversion κ. The volatility of the mean reverting process is σ. The third term of Equation (3) is the Poisson process with intensity λ, if we have a jump is size is normally distributed with mean μ_j and volatility σ_j. dW_t is the increment to a standard Wiener process and dq_t is a Poisson process such that $dq_t = 1$ with probability λdt and $dq_t = 0$ with probability $1 - \lambda dt$. dW_t and dq_t are independent.

Equation (4) below shows a discrete version of the mean reverting part of Equation (3).

$$(\alpha - \kappa X_t)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_t \qquad (4)$$

where ε_t stands for a random sample from a N(0,1) distribution.

Using the maximum likelihood estimation, as described in the Appendix A, we obtain the following values shown in Table 3 for the parameters of Equation (3):

Table 3. Parameters of stochastic equation

Parameter	Value	95% confidence interval
α	6.6910	5.7129 – 7.6783
κ	217.3689	203.7533 – 231.2784
μ_j	-0.0352	-0.0443 – -0.0261
σ	0.4900	0.3945 – 0.5342
σ_j	0.0959	0.0900 – 0.1014
λ	188.6669	165.8342 – 211.7488

Table 3 shows that the logarithm of the stochastic part of the number of flights tend to the level α/κ=6.6910/217.3689=0.03081 in the long term, that is, towards a value very close to zero. This variable deviates daily from equilibrium values with volatility equal to σ=0.49. In addition, there are negative jumps (as shown in earlier Figure 5), because this μ_j=-0.0352, when there is a jump, it is normally distributed with volatility σ_j=0.0959. For a given day dt=1/365, there is a probability λdt=51.7% chance of a jump, these jumps are expected to be of small size. Note that, the mean reverting part is compensated with the jump part to get an expected mean value equal to zero.

In addition, the confidence intervals of the correlation have been calculated following Ugarte et al. (2008) and are shown in Table 4 below. These values shows than although the correlation is negative and small it is scientifically different to zero.

Table 4. Confidence Intervals of the Correlation

Lim-Inf (95%)	Lim-Sup (95%)	Lim-Inf (90%)	Lim-Sup (90%)
-0.1495	-0.0580	-0.1424	-0.0655

In the case of air traffic, it is reasonable to assume that there is a certain limit to the number of flights an air space can manage and therefore that the increase in the simulated values will be lower.

Also note that in this case we have two different volatilities: one for the mean reverting process and a second one for the jumps or the Poisson process That is, after certain point, the closer to the maximum number of flights possible (or the limit of the air space management), the lower volatility may be expected. In order to test this assumption, we have estimated the historic correlation between the deterministic part and the stochastic part, where we have eliminated the jumps using three volatility criteria, that is the jumps are identified if there are larger that thrice the historic volatility of the stochastic part. The process is repeated until there is no jump because the volatility can change in each step. We obtain a correlation of -0.1036 between the logarithm of the deterministic part and the logarithm of the stochastic part. This calculation shows that the higher the number of expected flights (deterministic part) the lower the volatility of the stochastic part because the number of flights is reaching a certain limit. That is, a very high number of expected flights is associated with a lower stochastic volatility because there is a maximum number of flights, as sum of deterministic and stochastic parts, that cannot be exceeded at a certain time, so the assumption is right.

The correlated numbers are calculated according to the Equation (5) below.

$$v_1 = x_1\ ;\ v_2 = \rho_{1,2}x_1 + x_2\sqrt{1-(\rho_{1,2})^2} \quad (5)$$

Correlation is simulated by obtaining samples v_1 (normalised logarithm of deterministic part) and v_2 (normalised stochastic part) using equation 5, where x_1 and x_2 are two independent samples of logarithm of deterministic part and stochastic part, which are normalised in advance by subtracting the means of their distribution and dividing by the standard deviation of that distribution.

We use these values to simulate the air traffic for the year 2019. In order to do so we, first, simulate the stochastic daily part and, second, the deterministic one including daily part with the seasonality (annual and semi-annual), trend, weekend, and constant components together. For this purpose, we run 20,000 simulations for 2019 which account for a total of 365 days.

Figure 7 below shows the logarithm of historic number of flights and the simulated path of number of flights, including their deterministic part (historical and estimated). The same is done for the historic traffic and one simulated path. For these simulations we argue that the jumps are always negative as it can be observed in real data. There are a minimum traffic of 3,370 flights in 2014 and a maximum of 11,024 in 2018, and this behaviour was increasing in the last years with an annual mean rate of 2.96%. Note that as the real behaviour is only one of the paths in the simulation exercise, with a high number of simulations (for instance, 50,000 simulations), it is possible obtain higher values.

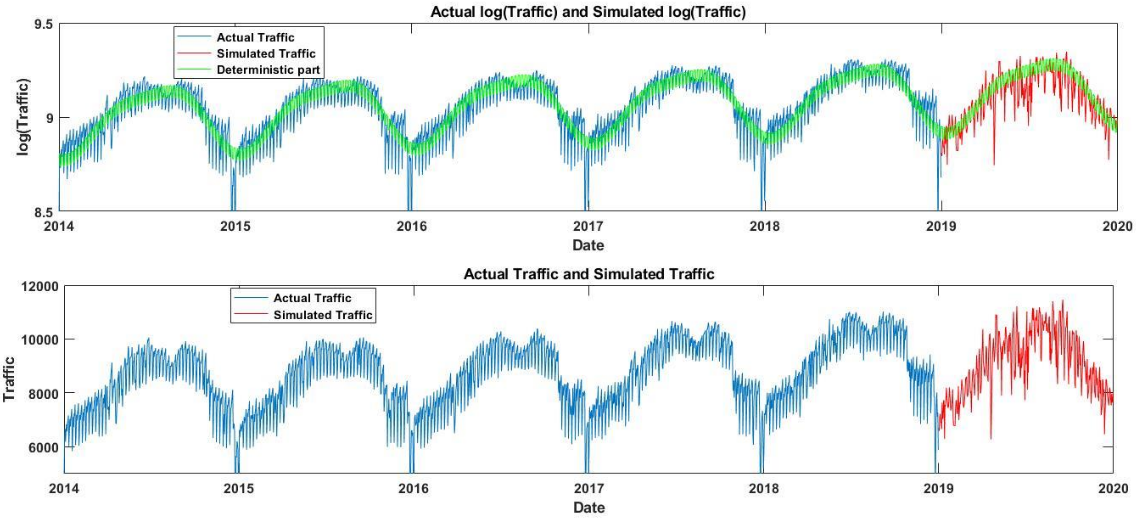

Figure 7. Historic number of flights and a simulated path of daily flights

Using the same number of 20,000 simulations for 2019 we obtain the distribution of daily traffic simulation shown in Figure 8 where the mean is 9,060 with a 95th percentile of 10,923 flights and the average of the 5% of the worst cases (upper tail) is 11,764 flights. This is the so-called Expected Shortfall (ES(95%)) and shows that in the 5% of the most extreme situations, the average number of flights will be 11,764. This is a very interesting indicator frequently used for risk and/or uncertainty management. See Abadie and Chamorro (2013) for additional clarifications. 20,000 simulations are done, each simulation is a path of possible values for 2019, the deterministic part is always the same. To the deterministic part, a stochastic part is added, the values of each stochastic part are different because the generated random samples are different. Using Monte Carlo simulation, it is possible represent the corresponding probability distribution.

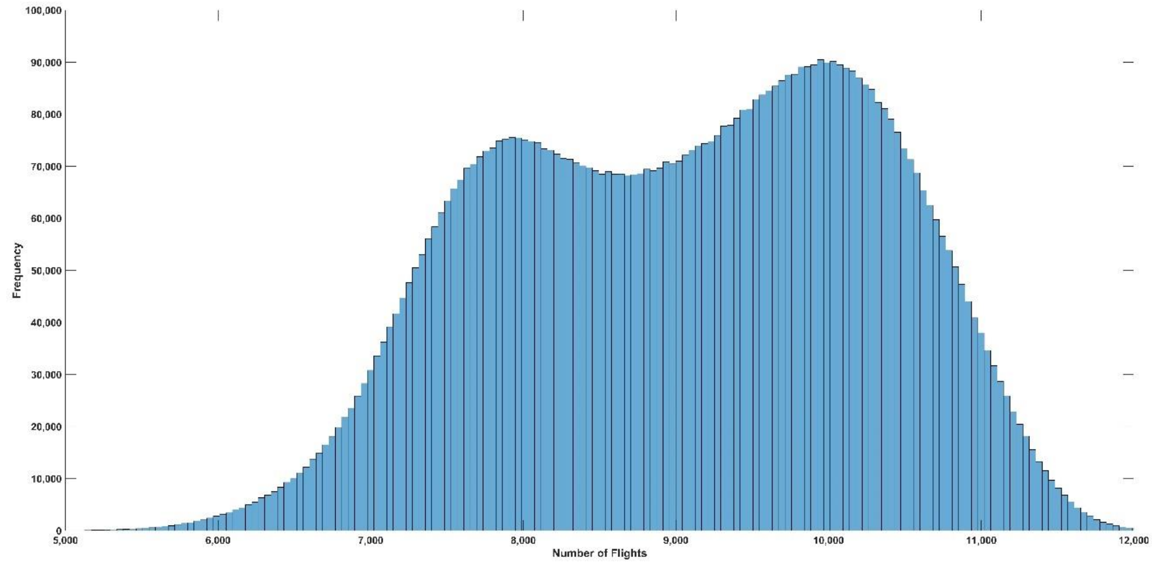

Figure 8. Distribution simulation flights traffic year 2019

Now, we can analyse if the proposed and calibrated model can be used for predicting the number of flights for future years. Note that the mean of simulated results is the deterministic part because the mean of stochastic part is zero.

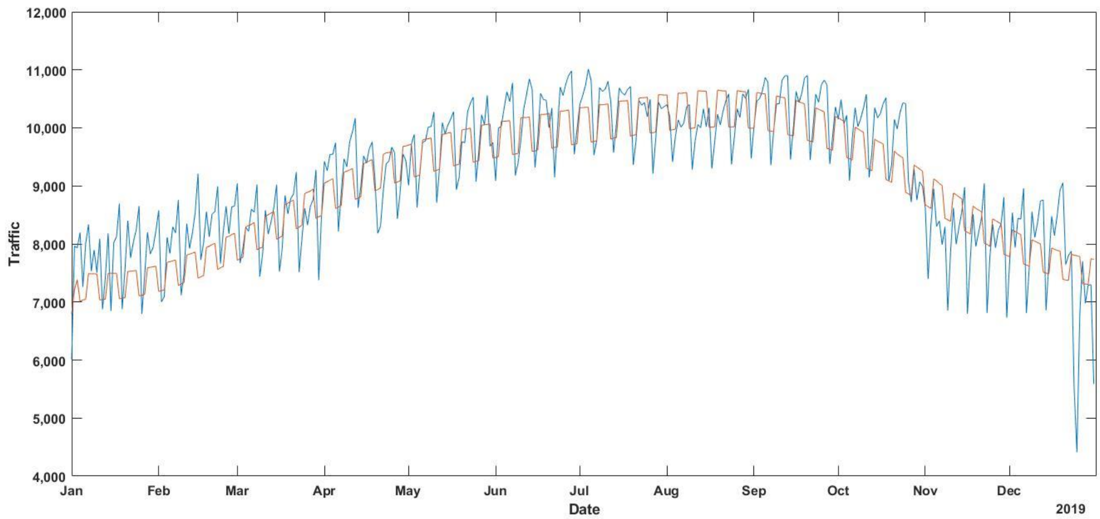

Figure 9. Real Flights 2019 and mean of simulated flights for this year

While the total annual number of flights in 2019 was 3,348,085, our simulation estimated a mean value of 3,306,829. Both these figures are relatively close to each other, showing a less than 1.25% difference among them and thus illustrating the goodness of the model. The goodness of fit between the expected simulated daily traffic values and the actual are calculated using the R-squared with a result of 0.73. Figure 9 shows this goodness of fit. Also note that the number of flights in 2019 was very similar to 2018. The difference was only 0.3%. However, this difference was much higher during the years 2014 to 2018, ranging from 1.6% to 4.2%. In Appendix B we illustrate the performance of the stochastic approach with simulations for 2017 and 2018 and compare them with more basic approaches. This proves that the proposed methodology is adequate, and this is especially true the greater the volatility is.

3.2 The Tobit model of delays

Turning now to the estimation of the delays and its connection with the number of flights, one can start by showing real data on delays as displayed in Table 5. Note that during the five years analysed the average delay was 0.760 minute per flight, while these figures were significantly higher during some specific years, such as 2017 and 2018.

Table 5. Real traffic and delays

	Traffic	Delays (minutes)	Delays per flight
2014	2,990,197	1,224,205	0.409
2015	3,039,298	905,337	0.298
2016	3,118,818	1,581,331	0.507
2017	3,222,709	2,635,383	0.818
2018	3,359,668	5,605,831	1.669
5 years	15,730,690	11,952,087	0.760

Of a total of 1,826 days there are 95 (5.2%) without any delay and 1,731 (94.8%) days with delays. Also note that the observed data is left-censored as no negative values are expected. We, thus, select the Tobit model as the appropriate one to analyse this dataset (Amemiya, 1984). With the use of this model we can connect the number of flights to the number of delays. Note that the Tobit model fits well with a situation in which, below a certain level of air traffic, no delays are expected, while, once a threshold (congestion level) is reached, delays increase proportionally to the congestion level.

The Tobit model is formulated as shown in Equation (6) below,

$$y_i^* = \beta' \mathbf{x}_i + \varepsilon_i \qquad (6)$$
$$y_i = 0 \text{ if } y_i^* \leq 0$$
$$y_i = y_i^* \text{ if } y_i^* > 0$$

where x_i are the set of independent variables (in our case only is the air traffic and a constant) and the constant term, y_i^*, is the latent variable that is unobservable and ε_i is a normally distributed error term. A similar approach is explained in Abadie & Goicoechea (2019b).

When the latent variable is negative or zero, the Tobit model predicts no delays, while, when the latent variable is positive, the model is about the expected delays. The Tobit model results calculated using Stata with the 1,826 observations are shown in Table 6 and allow us to estimate when delays are to be expected and the number of minutes of these.

Table 6. Tobit regression

					Number of obs =	1,826
					LR chi2(1) =	582.61
					Prob > chi2 =	0.0000
Log likelihood = -18,219.485					Pseudo R2 =	0.0157
Delays	Coefficients	Std. Err.	t	P>|t|	[95%	Interval]
Traffic	4.351592	0.1697535	25.63	0.000	4.018661	4.684524
_cons	-31,344.22	1,484.457	-21.11	0.000	-34,255.63	-28,432.8
/sigma	8,701.598	147.9105			8,411.507	8,991.69
95	left-censored observation at Delays<=0					
1,731	uncensored observations					
0	right-censored observation					

The likelihood ratio (LR) chi-square with p-value=0.0000 informs us that the Tobit model is significantly better than an empty model. The coefficients are statistically significant. We have a 4.351592 increase in y_i^* (delays if $y_i^* > 0$). The statistic /sigma with a value of 8,701.598 is the estimated standard error of the regression. It is expected some delays when traffic is greater than

7,203 (Figure 10). Figure 10 below shows the value of y_i depending on the traffic and the real values corresponding to five years (2014-2018). We can see that the volatility increases with traffic.

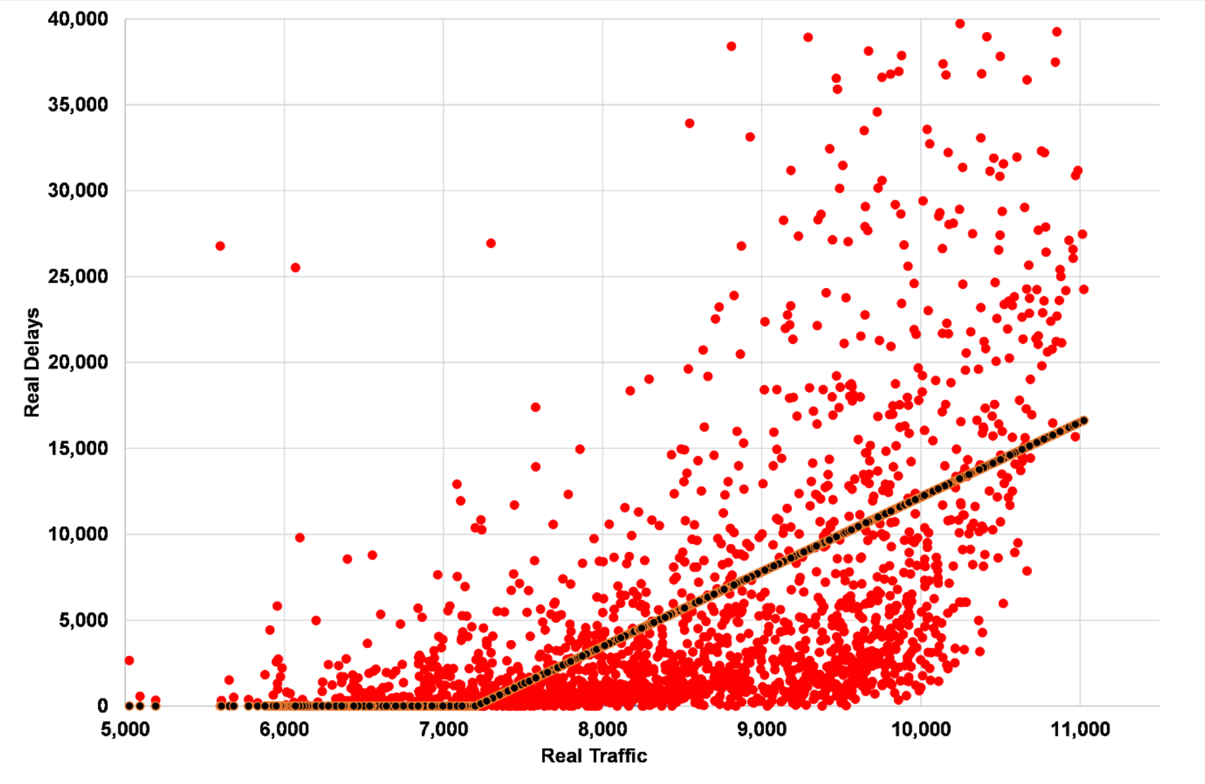

Figure 10. Effect of actual traffic on expected daily delays and actual delays (2014-2018)

Table 7. Real and predicted delays

		Predicted	
		Delay	No Delay
Real	Delay	1,524	207
	No Delay	40	55

Table 7 shows than in 1,579 cases (1,524+55), that is in an 86.5% of cases, the Tobit model correctly predicts the situation with delays and no delays.

Table 8 shows a Monte Carlo simulation in the base calculated case for the year 2019. This Table also includes a sensitivity analysis changing the volatility. In the base case, the annual delays mean is 3,496,903 minutes with a 95th percentile of 3,780,744 and the mean of the 5% of worse cases is 3,853,918 (ES(95%)). In this Table 8 we can also see the effect of changes in the volatility where the mean changes because it is a censored model on the left.

Table 8. Sensitivity Daily Delays to Tobit volatility in 2019

Tobit Model	Volatility	Mean	95th	ES (95%)
	150%	3,973,036	4,336,948	4,433,062
5 last years	Calculated	3,496,903	3,780,744	3,853,918
	50%	3,146,877	3,344,699	3,394,650

Figure 11 shows the results of the base case for the first five simulations. Of course, it is not possible to illustrate all 20,000 simulations jointly in a single Figure. However, as all simulations are equally probable, we illustrate just five of them.

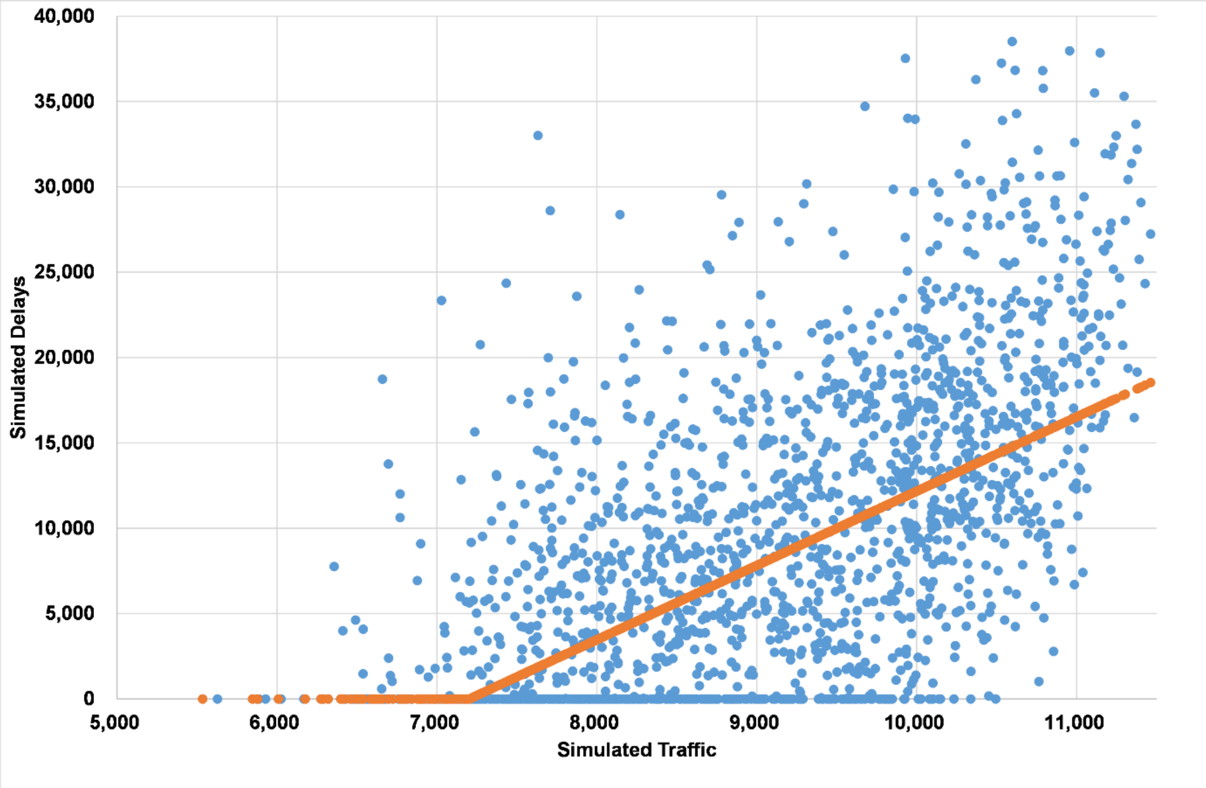

Figure 11. Simulation of daily Delays (five first simulations for 2019)

The proposed model predicts the delays as a function of the number of flights. Other explanatory variables not considered in this model as well as the volatility inherent in this relationship cause the prediction not to agree exactly with reality, but to be reasonably close in most cases (see Figures 10 and 11). This statement can be shown by comparing the parameters of the simulations and the relevant values of parameters calculated with actual. This is included in Appendix C.

Figure 12 shows the distribution tail of simulated traffic delays. One can see that the frequency of daily delays declines sharply the closer we get to relative high values of delays.

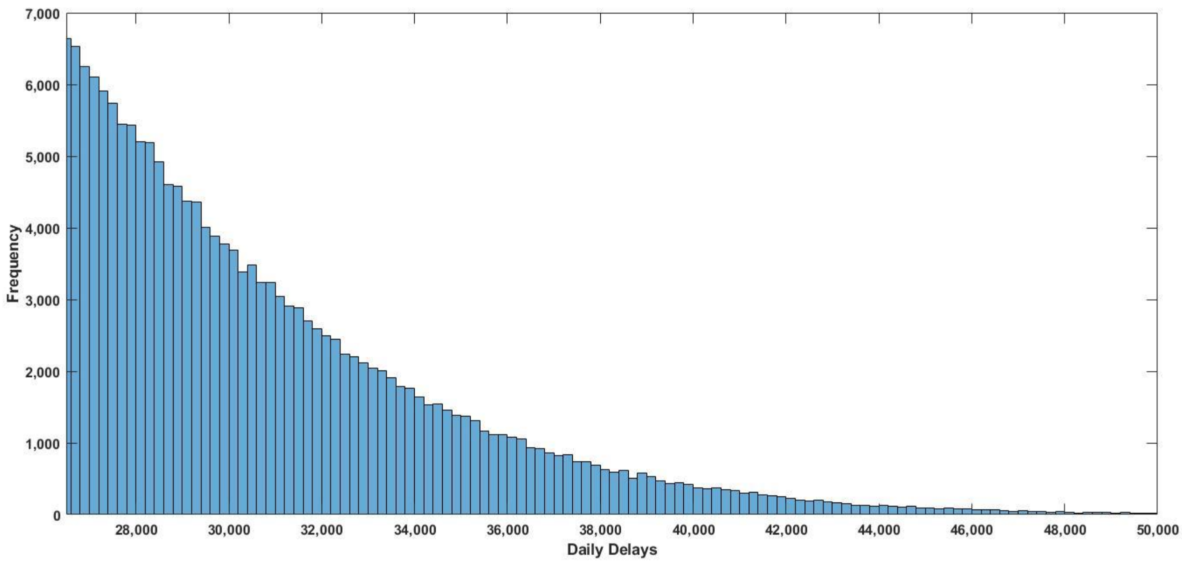

Figure 12. Simulation tail of daily delays for year 2019 (all daily simulations)

However, note that carefully selecting the period for the estimation may significantly change the results. That is, whether the years used for calculations reflect a tendency and whether what happened in 2018 represents an exception remains to be answered. This acknowledgement suggests that expert opinion has to be carefully incorporated in this analysis, to co-decide on the reasonable number of years to be used to estimate a distribution of delays. If only 2018 data was to be used, values may differ substantially. This caveat is not particular for this method but inherent to any forecasting exercise. In any case, it needs to be clearly acknowledged.

The distribution of 10,000 simulations for 2019 are shown in Figure 13 below.

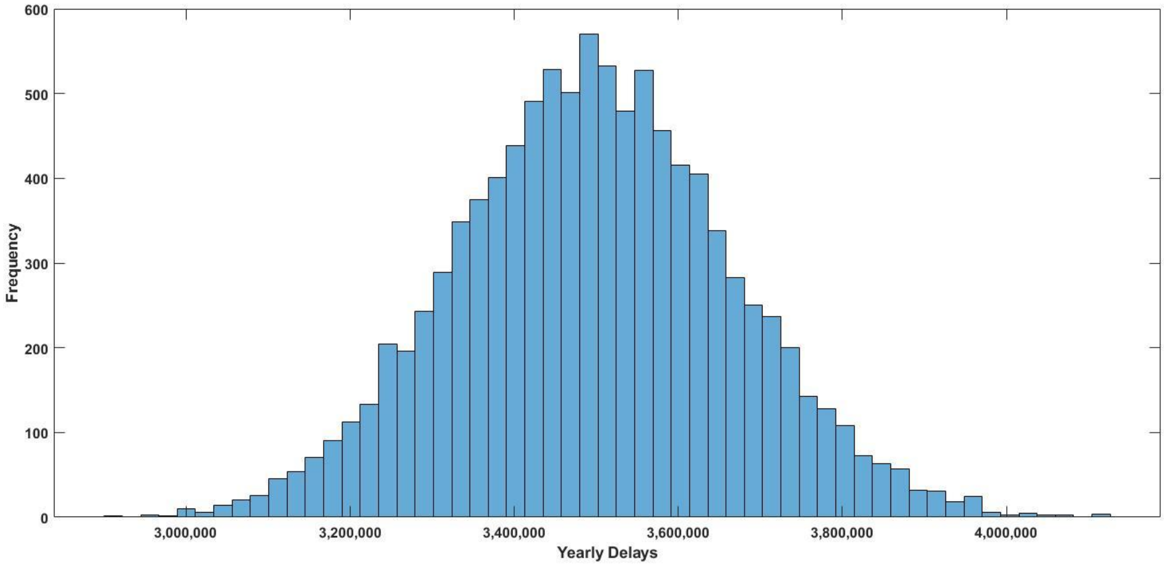

Figure 13. Simulation of yearly delays in 2019

3.3 The cost model of delays

The first attempt to estimate the monetary cost of one minute of ATFM delay, as incurred by the Airliners in Europe, was made by the Transport Studies Group from the University of Westminster (2004) where such delays were measured relatively to the last filed flight plan (Cook, 2004). Successive revisions and updates of the study were published (Cook & Tanner 2011, 2015; Performance Review Unit 2019).

By way of summary, the network average cost of ATFM delay was 102 euros/minute (in 2017 values). Many have been tempted to use this value for back of the envelope estimates of cost of delays multiplying by the total minutes of delay. However, there two important points to be considered: (1) not all flights are subject to the same amount of delay, so it becomes crucial to determine the distribution of these delays and how this affects the total cost; and (2) that the average costs of delay may also change depending on the amount of minute delay. In this paper, considering the information available, we assume that delays can be ranged in three delay classes as explained later, these are: from 0 to 15 minutes, from 15 to 60 minutes and from 60 minutes upwards.

Cost of delays using a mean of 102 €/ minute

If we now use the average figure of 102€ per minute delay and apply for the distribution of delays, we can estimate the cost distribution displayed in Figure 14 for the year 2019. This offers a much complete picture than back of the envelope calculation mentioned above. However, cost ranges have not yet been included in the analysis. This will be shown below.

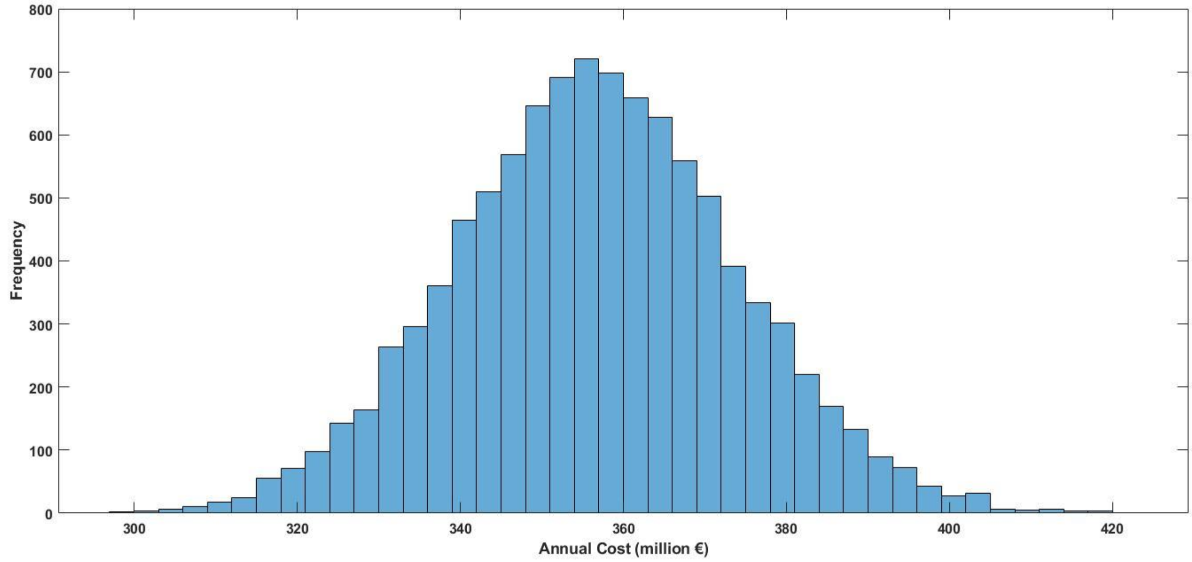

Figure 14. Simulation of yearly costs in 2019

In order to illustrate the sensitivity of the annual cost to changes in the volatility of the Tobit model we shown the values in Table 9 below. These are a linear transformation of Table 8 values for the three-year case. The mean values may be 10% lower or 14% higher, as well as the percentile 95 and the ES (95%) with the variation of 50% in the estimated volatility.

Table 9. Sensitivity Annual Costs to Tobit volatility 2019 (millions €)

Volatility	Mean	95th	ES (95%)
50%	320.98	341.16	346.25
Calculated	356.68	385.64	393.10
150%	405.25	442.37	452.17

Cost of delays depending on delay values

If we now, in addition to the distribution of delays, wish to obtain values resulting from using different costs for different delays ranges the following can be done. First, based on the average cost per delay range from Cook & Tanner (2011), we can calculate the cost per minute by dividing the average cost by the midpoint of the range, as shown in Table 10.

Table 10. ATFM delay ranges and weighted costs – total and per minute – (in 2010 €) for ten delay ranges

Delay range (min)	01-04	05-14	15-29	30-59	60-89	90-119	120-179	180-239	240-299	300+
Average delay (min)	2.5	9.5	22	44.5	74.5	104.5	149.5	209.5	269.5	300
Average cost (€)	32	210	870	3,870	11,940	25,560	39,710	53,220	70,450	80,270
Cost per minute (€)	12.8	22.11	39.55	86.97	160.27	244.59	265.62	254.03	261.41	267.57

Source: Cook and Tanner (2011), Table J5 (2010 values)

These are 2010 values, while we need 2017 values. Considering that the network average cost of ATFM delay was 102 euros/minute in 2017 values, as explained above, and 81 euros/minutes in 2010 values, this means that there has been an increase of 25.926%. By applying this percentage increase to the values of the last row in Table 10, an approximation of ATFM delay cost per minute in 2017 values (Table 11) can be calculated.

Table 11. ATFM delay ranges and weighted costs per minute per minute (in 2017 €) for ten delay ranges

Delay range (min)	01-04	05-14	15-29	30-59	60-89	90-119	120-179	180-239	240-299	300+
Cost per minute (€)	16.12	27.84	49.80	109.51	201.82	308.01	334.48	319.89	329.18	336.94

As we now need to narrow these ranges to only three, we can aggregate these values (adding the cost for each range and calculating the average) and obtain the values shown in Table 12:

Table 12. ATFM delay ranges and weighted costs per minute (in 2017 Euros) for three delay ranges

Delay range (min)	0-15	16-60	60+
Cost per minute	21.98	79.66	305.06

Once cost per minute has been estimated, the next step consists of multiplying the number of minutes of delay considering the full distribution for the three delays ranges by the cost of them. This allows us to obtain the total cost of delays, for illustrative purposes these costs have been calculated for the years 2017 and 2018 only. This is displayed in Table 13.

Table 13. Yearly real delays and costs

	Delays (minutes)	Total Cost (€)	Cost/minute (€)
2017	2,635,383	217,946,663	82.70
2018	5,605,831	466,288,228	83.18

Using the information of Table 12, we can also estimate a cost function to be used for calculations. The function should meet the following theoretical conditions:

a) Only make sense for positive delay values;
b) Goes through the origin;
c) Must be a growing function, that is, the first derivative should be positive; and
d) As values are expected to grow rapidly with delays, the second derivative should also be positive.

The function proposed (7) is as follows:

$$Cost = aDelay^{b} \tag{7}$$

which is equivalent to (8):

$$\ln(Cost) = \ln(a) + b\ln(Delay) \tag{8}$$

Based on the calculated data covering the years 2017 and 2018 (by removing cases with zero delays), the Ordinary Least Squares (OLS) estimation is used, obtaining the following results in Table 14.

Table 14. Cost model estimations

Model: OLS, using observations 1-727
Dependent variable: Ln(Cost)
Heteroscedasticity-consistent standard errors (HC1)

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
ln(a)	4.00799	0.0534297	75.01	<0.0001	***
b	1.03526	0.00594817	174.0	<0.0001	***

Mean dependent var	12.93588	S.D. dependent var	1.495920
Sum squared resid	35.98649	S.E. of regression	0.222793
R-squared	0.977849	Adjusted R-squared	0.977819
F(1, 725)	30292.10	P-value (F)	0.000000
Log-likelihood	61.03377	Akaike criterion	−118.0675
Schwarz criterion	−108.8897	Hannan-Quinn	−114.5260

Coefficients are significant and the R-squared value is very high (0.977849). Graphically (Figure 15), it can be shown that the function fit very well with real data on cost of delays.

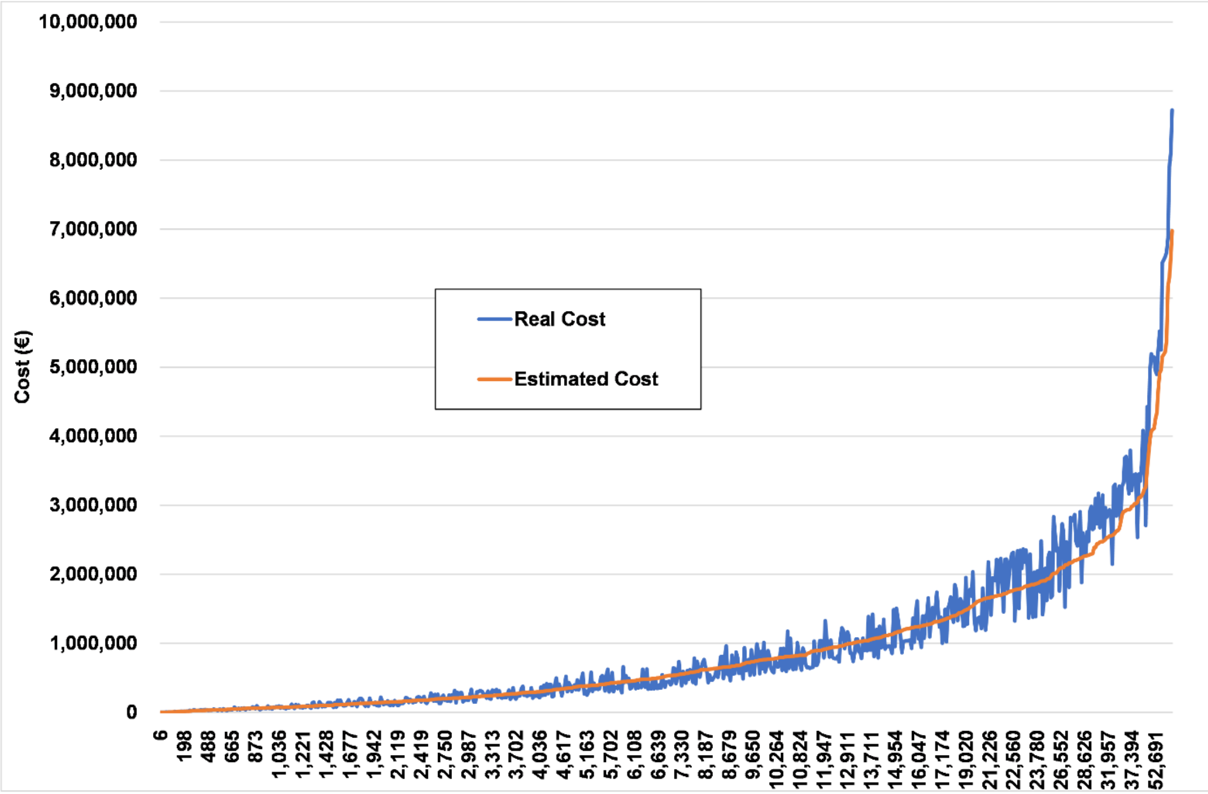

Figure 15. Real Costs and expected costs using the function estimated

Finally, Table 15 below shows the estimated cost (mean, 95th percentile and ES(95%)) for the case of higher or lower volatility than the estimated with the Tobit model. Note that now that the real data for 2019 is available, with a real cost of 362.20 million euros, one could compare this with the mean values estimated by the model. In this case, we can see that the real cost in 2019 is really close to the mean value estimated in the case of greater volatility (See Table 15).

Also note that the mean values may be more than 22% higher or 16% lower, the percentile 95% maybe higher 23% or 17% lower and the ES (95%) can go up to 23% higher or 18% lower with the variation (up or down) of 50% in the estimated volatility.

Table 15. Annual 2019 cost sensitivity to Tobit volatility (millions €)

Volatility	Mean	95th	ES (95%)
50%	236.36	257.59	262.84
Calculated	280.28	310.54	318.67
150%	341.72	382.63	393.46

Figure 16 displays the estimated full distribution of annual costs of delays for the year 2019.

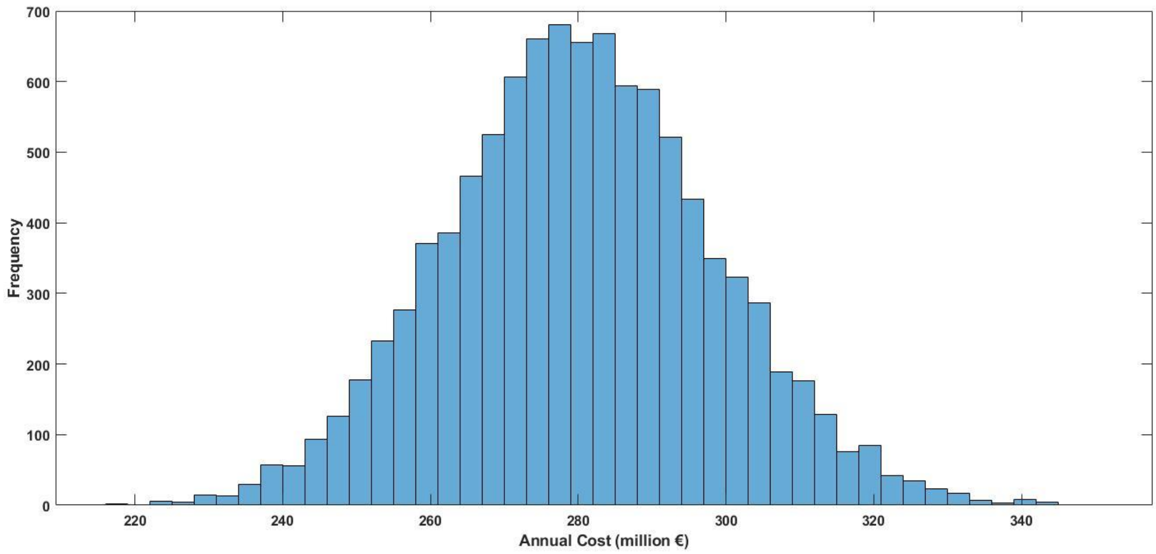

Figure 16. Simulation of yearly costs in 2019 with calculated volatility

Risk measures such as ES(%) can be used to fix a certain threshold of low-probability high-impact events that ANSP wishes to be protected from (or for which to be specially prepared). This information can help ANSP to better tailor their risk appetite and plan to avoid a certain given level of economics costs of delays (or risk). ES is a well-known risk measure in financial, energy and climate economics (Artzner et al. 1999; Abadie & Chamorro 2013; Galarraga et al. 2018). For instance, DFS could decide that for the year 2019, the maximum acceptable level of risk will be to allow that for 95% of the cases the average cost is never higher than 319 million €. That is, DFS may wish to fix the risk to the ES (95%) for 2019. This is the method proposed in Galarraga et al. (2018) to define the risk appetite and can analogously applied here. Having defined this threshold of economic risk ES(95%)) which is equivalent to 319 million €, the operative goal should be set to a level that guarantees that the number of delays is not higher than 3.8 million minutes (see Table 7). DFS could thus plan its level of services to ensure that the delay target is very unlikely to be exceeded. Of course, with the information estimated by this model any given level of percentage can be used to calculate the corresponding ES(%) to better tailor the risk a given ANSP is willing to accept.

4. Conclusions

This paper has proposed and applied a sophisticated stochastic modelling method to simulate air traffic in the German air space for the year 2019 using data from the period 2014-2018. This is later used to, first simulate air traffic delays for the year 2019 and, second the economic costs associated to these air traffic delays for the same year. The combination of three models allows us to display a simulation of the full distribution of costs of delays (at daily basis) for the year 2019. That is, the detailed distribution of daily traffic, daily delays and consequently daily costs. This information is much more detailed that the regular forecast available in the sector. The goodness of fit of the air traffic estimates for 2019 results in an R squared of 0.73, which is fairly good.

As far as we are aware, this is the first attempt in the literature to offer a complete picture of how the costs of delay may behave, considering both the distribution of delays as well as different costs for different costs ranges. Note that the method does not only predict the annual values, but it also provides estimates for the day to day traffic, delays and costs, taking into account both deterministic and stochastic parts of the behaviour (i.e. considering the certain and uncertain movements). Moreover, the proposed methodology generates probability distributions for all these. Unfortunately, hourly date for both traffic and delays was not available to enhance even more the quality of the estimates. This is a caveat of the study presented here. But it is more the consequence of the difficulty to access hourly data than a problem of the method that could be easily adapted to work with a much higher granularity of the data.

The information provided in this paper may be of great use for decision-makers to consider accordingly as it is a very sound method to support decision making in the context of uncertainty. Therefore, it allows the decision maker to be fully aware of what the implications of different delays may mean in terms the total costs at daily basis. In fact, having the full distribution allows to calculate both the 95% percentile as well as the ES (95%) that are well-known measures of economic risks and have been extensively used in financial economics and are being applied in energy and adaptation economics more recently. These are quite powerful tools to support decision making under uncertainty. The measures can be directly used to tailor the risk appetite and plan the resources needed to avoid certain costs ranges to be exceeded. Even to undertake stress tests of what would happen in the traffic and delays behave in different (unlikely) ways.

The calculations suggest that for an average cost of 82.70€ per minute in 2017 and 83.18€ per minute in 2018, overall mean delay costs for 2019 may be up to 280 million €, while in the 5% worst (unlikely) cases the average damage can be as high as 319 million € only in Germany. These values can well be used to define a base-line of services needed to be operative to avoid the economic costs estimated and this guide ANSP planning processes.

Acknowledgements

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APPENDIX A: Stochastic Delay Model Estimations

We can represent the density function of X_t given X_{t-1} as in Equation (A.1)

$$f(X_t|X_{t-1}) = \lambda\Delta t N_1(X_t|X_{t-1}) + (1-\lambda\Delta t)N_1(X_t|X_{t-1}) \quad \text{(A.1)}$$

With probability $\lambda\Delta t$ we have a jump and then applies the Equation (A.2):

$$N_1(X_t|X_{t-1}) = \frac{1}{\sqrt{2\pi(\sigma^2+\sigma_j^2)}} e^{-\frac{(X_t-\alpha\Delta t-(1-\kappa\Delta t)X_{t-1}-\mu_j)^2}{2(\sigma^2+\sigma_j^2)}} \quad \text{(A.2)}$$

With probability $(1-\lambda\Delta t)$ there is no jump and then applies the Equation (A.3):

$$N_2(X_t|X_{t-1}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(X_t-\alpha\Delta t-(1-\kappa\Delta t)X_{t-1})^2}{2\sigma^2}} \quad \text{(A.3)}$$

The parameters $\theta = \{\alpha, \kappa, \sigma, \lambda, \mu_j, \sigma_j\}$ can be calculated minimizing the negative value of the log likelihood functions as Equation (A.4):

$$\min_{\theta} - \sum_{i=1}^{i=T} \log(f(X_t|X_{t-1})) \quad \text{(A.4)}$$

subject to:

$(1-\kappa\Delta t) < 1$

$\sigma > 0$

$\sigma_j > 0$

$\kappa > 0$

$0 \leq \lambda\Delta t \leq 1$

APPENDIX B: The Performance of Stochastic Models Versus Basic Approach

To show how the proposed stochastic model performs, we have estimate applying the same approach: (1) the daily traffic for 2018 based on data from 2014-2017 and (2) the daily traffic for 2017 based on data from 2014-2016.

For the 2018 estimation there are 1,461 daily items with information on air traffic and delays in Germany. Table B1 shows the deterministic parameters that are used to determinate the expected values.

Table B1. Deterministic parameters calculated with daily flights 2014-2017

Parameter	Value	95% confidence interval
β_1	-0.0532	-0.0592 - -0.0473
β_2	-0.1592	-0.1650 - -0.1534
β_3	-0.0051	-0.0109 - 0.0008
β_4	-0.0410	-0.0468 - -0.0352
β_5	0.0233	0.0197 - 0.0270
β_6	-0.0660	-0.0743 - -0.0577
β_7	9.0145	9.0054 - 9.0236

Table B2 shows the stochastic parameters as well as its confidence intervals. Note that the stochastic part it does not intervene in the calculation of the expected values, but it does in their distributions.

Table B2. Parameters of Stochastic Equation

Parameter	Value	95% confidence interval
α	6.165177	5.153 - 7.1773
κ	222.9142	236.6343 - 209.1942
μ_j	-0.03039	-0.0391 - -0.0217
σ	0.447438	0.3762 - 0.5088
σ_j	0.095894	0.0897 - 0.1017
λ	200.4987	178.5581 - 222.4394

Figure B1 shows, the actual number of flights 2018 and the expected flights for the first simulated path in 2018 using the proposed model.

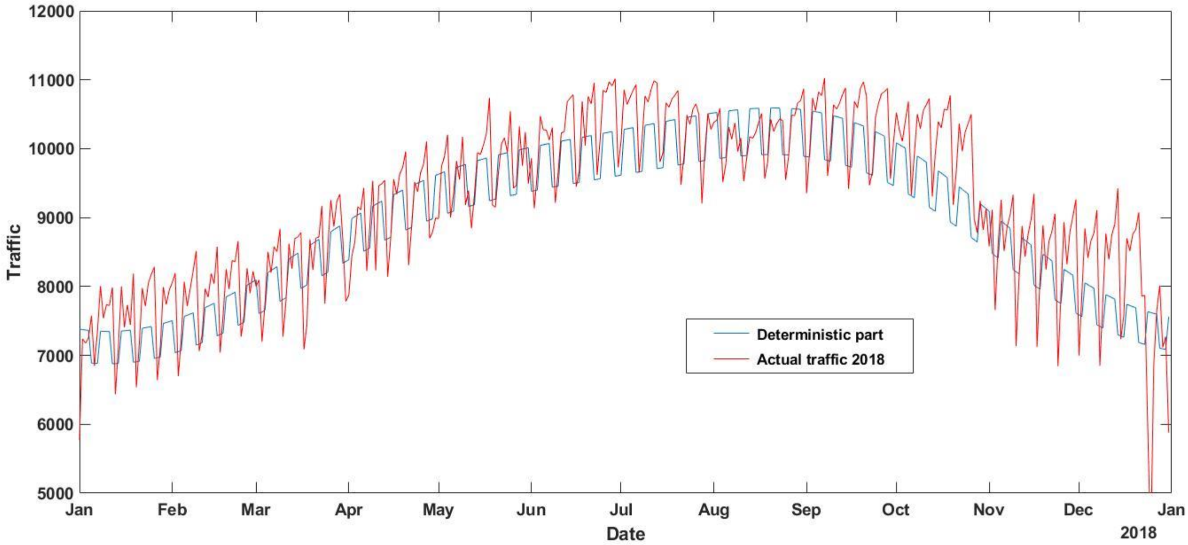

Figure B1. Actual Flights 2018 and mean of simulated flights for this year

Figure B1 shows, the daily actual number of flights of year 2018 and the expected figures for the same period (deterministic part). It is important to note that the forecasted values are very close from the actual values at daily basis. This is a very important feature of the modelling effort, that is, it is not only the total annual estimated data, which is quite reliable, but also the daily behaviour. The total number of flights predicted for 2018 with this model is 3,266,322.

If we now perform the same exercise for the year 2017 (using the data for 2014-2016), we obtain that the estimated number of flights is 3,144,460. Table B3 shows the values of deterministic parameters calculated with this period as well as the confidence interval.

Table B3. Deterministic parameters calculated with data on daily flights 2014-2016

Parameter	Value	95% confidence interval
β_1	-0.0532	-0.0592 - -0.0473
β_2	-0.1592	-0.165 - -0.1534
β_3	-0.0051	-0.0109 - 0.0008
β_4	-0.0410	-0.0468 - -0.0352
β_5	0.0233	0.0197 - 0.027
β_6	-0.0660	-0.0743 - -0.0577
β_7	9.0145	9.0054 - 9.0236

Let us compare the estimated (or forecasted) values for the years 2018 and 2017 using the stochastic method with the actual data and, also with a basic approach of using earlier year's data as the forecast. This is shown in Table B4 below.

Table B4. Actual and forecasted number of flights

Years	Forecasted Flights Stochastic Model	Forecasted Flights Basic	Actual Flights	Difference (%) Stochastic-Actual	Difference (%) Basic-Actual
2017	3,144,460	3,118,818	3,222,709	-2.4%	-3.2%
2018	3,266,322	3,222,709	3,359,668	-2.8%	-4.1%

The calculations clearly show that when the stochastic method is applied, the difference between the forecasted and the actual one is significantly smaller (between 25% to 32%) than when the basic approach is used. That is, the stochastic model predicts better than just using the basic approach of using the year before as an indicative of what the year will look like. This is also true if one takes into consideration daily behaviour. And more importantly, note that the greater the volatility the better the stochastic model performs with respect to more basic approaches (Hull, 2014).

APPENDIX C: Real Data and Simulations

Note that while Figure 10 depicts the actual data 2014-2018, Figure 11 is obtained from the five first simulations for 2019. Therefore, the calculations represented in Figure 11 are affected by two factors:

a) Different values in the number of flights as compared with actual data 2014-2019 because we are using estimated values for 2019.

b) Only five of the 20,000 simulations are shown because it is not possible to depict all of them in a single "readable" figure. The simulated delays 2019 are calculated using the parameters of period 2014-2019.

If we now estimate the values of parameters calculated with actual data shown in the Table below, we can show that in both cases the predicted delays are zero for flights values lower than 7,203 with a slope of 4.351592 for larger values. This is the case of a mean value of the 20,000 simulations. For simulation purposes the same volatility was used with actual data.

Delays	Coefficients
Traffic	4.351592
_cons	-31,344.22
/sigma	8,701.598