

**IKERLANAK**

**THE LANGUAGE GAME:  
A GAME-THEORETIC APPROACH TO  
LANGUAGE CONTACT  
Part I: Static Analysis**

by

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A Game-Theoretic Approach to Language Contact\***

**Part I: Static Analysis**

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## ABSTRACT

We study a society inside which two official languages, the majority language  $A$  and the minority language  $B$ , are in contact and compete for the same social functions. We propose a non-cooperative game to capture some features of this competitive situation. In the game, there are two types of players: the bilingual one who speaks both  $A$  and  $B$  and the monolingual one who speaks only  $A$ . The information about which type is each player is private.

A real life situation captured by the game is that in many interactions bilingual players must decide under incomplete information about which language to use. One implication of this information structure is that while  $A$  satisfies the main properties of a public good,  $B$  does not. Another implication is that it may have dangerous consequences on the language diversity of the society. We show that in many equilibria bilingual players fail to coordinate in their preferred language and end up using the majority language  $A$ .

## 1. Introduction

Sociolinguists remind us that (i) economic and/or political power is always exerted in a specific language and (ii) since language is such a central ingredient in human culture and cultural identity, we tend to convert language into a very important issue, more so when it is felt that the society's perceived "own" language is under threat : "The western world has had a love affair with its own languages for over two centuries. Because of this love affair (the seeds of which the power and prestige of the West has also sown throughout much, if not most, of the world), it is easy to overstate the importance of language in human social and cultural affairs" (Fishman(2001)).

An issue frequently studied in the sociology of language is the minority language maintenance in the context of powerful forces operating in modern societies that push in the direction of minority language shift. The outcome of this process is that there are many languages under threat, many thousands are dying and many more will die in the near future. "Unfortunately, democratic regimes can be just as blind to cultural pluralism and to the needs of minorities for cultural recognition and support, as are autocratic regimes". (Fishman (1991)). The, approximately 6,000, existing languages of the world (about the difficulties to quantify the number of existing languages, see Crystal(2001)) were classified by Krauss(1992) into three categories: "(1) 20-50% of the total are "moribund languages" that children have stopped using as mother tongues, (2) 40-75% of them are "endangered languages" that children are still learning as native speakers but that are likely to become moribund by the end of the 21st century, and (3) 5-10% of them are "safe languages" that seem to be assured of their future existence. So, in the worst case, 95% of the languages spoken in the world today could be extinct or be on the way of extinction during the next century". (Nara (2000); see also Skutnabb-Kangas(2000)). India is an exception of this trend, a nation with many languages and a Constitution that designates no one language as the national language. The linguist Annamalai (2000) says that "the linguistic history of India has been one of addition of languages, and not their reduction".

Needless to say that the perception that your own language is under threat may have profound political implications. The Mysore Document (2000) stresses

the idea that “the preservation of linguistic diversity is necessary not only for ethical reasons but also as an input for human progress and development. Every language codifies a way of cognizing, experiencing and organizing knowledge of the world. Each way has the potential to correct any wrong step taken by “successful languages and cultures” which may be detrimental to humanity”. The first recommendation of the Mysore Document says that every State “shall ensure equal rights and opportunities to all linguistic communities for survival and development”.

We shall concentrate on societies inside which two official languages are in contact and compete for the same social functions (in education, research, government -executive and legislative wings-, administration of justice, business, banking and insurance regulations, information technologies, mass media, entertainment etc.). It is very common to observe in that type of societies that one of the languages has a relatively smaller social support than the other; one language is spoken by some, while the other one is known by all the members of the society and dominates both the private and the public domain. In those societies we would have a minority language community who is able to speak both languages and a majority who speaks only one language. It is easy to predict what would be the outcome of language competition under those circumstances. The weaker language will lose social functionality, unless its speech community takes some actions. Examples of this type of languages and societies abound. To name just a few: Irish in Ireland, Welsh in Wales and Gaelic in Scotland are minority languages in contact with English; Basque in the Basque Country is a minority language in contact with “big languages”, French and Spanish; Galician spoken in Galicia is in contact with Spanish; Frisian spoken in the province of Friesland in The Netherlands is in contact with Dutch; the Aboriginal languages of New Zealand and Australia are in contact with English, etc.

Hence, we have a relevant social conflict affecting many societies, the contact of languages which compete inside a society for the same social functions, with some languages and their related cultures and collective identities under threat or fighting for survival. The present paper models certain central features of this competitive situation, the contact of languages, by using non-cooperative game tools.

Language, and particularly the economic analysis of bilingualism and language policy, has been central in the works of Canadian economists, such as Breton (1998, 1999) and Vaillancourt (1996). Most of these works are a mixture of theory and applied economics, one of the purpose being the evaluation of minority language

policies, as in Grin and Vaillancourt (1999). Along this line, Grin (1996, 2003 ) shows how fruitful can be the economic approach to language; his work is an initial step in trying to introduce basic microeconomic tools in the study of language and facing the difficulties of defining the notions of supply, demand and price of a language (specific good).

Even though it is a fact that linguistic diversity and language contact is accompanied by conflict (some authors like Nelde (1997) emphasize that there is no language contact without social conflict, and others, like Grin (2003), emphasize that linguistic diversity tends towards conflict) it is surprising how little or null attention has received this issue from economic theory. By this we do not mean that language is not considered relevant to economic theory. Language appears mainly in relation with cheap-talk games, (see, for instance, Crawford and Sobel (1983) and Farrell (1993)) where what matters is that players have a unique system of communication or a unique labeling -and it does not matter which one- so that information is transmitted and messages are understood. But, as stressed by Pei-yu Lo (2005), the restriction imposed by the assumption that players speak the same natural language is not taken into account in cheap-talk games. Nevertheless, even when it is introduced the notion of language, the analysis usually deals with the *corpus of the language* (that is, the internal structure and functioning of a language) or, in a different setting, with *pragmatics* ("the study of factors that govern the choice of language -here language means certain words and sentences- in social interaction and the effects of our choice on others", Crystal (1987)), as it happens in the approach of Rubinstein (2000).

The present paper deals instead with the *status of the language* (the relative prestige and power -measured in terms of the size of the speech community, the degree of use in education, mass media, information technologies, etc...- of a language in a language contact situation), and it is in this sense that it is more related to the work of Pool (1986) and Selten and Pool (1991), although the issue studied here is of a different nature.

The Language Game models the interaction between two players with incomplete information. There are two types of players, the bilingual (i.e. the one who can speak both the majority language  $A$  and the minority language  $B$ ) and the monolingual ( i.e. the one who only speaks the majority language  $A$ ). We assume that  $A$  and  $B$  are in contact; they could be the official languages of the society, although this is not needed for  $A$  and  $B$  be in contact. The competitive situation between  $A$  and  $B$ , that we mentioned above, is embedded in apparently innocuous interactions, such as in a simple conversation. Suppose the interaction is between

a seller and a potential buyer and that it is located in a shop. Suppose too that none of the two players know the type of the other and that there are no signals (accents, speech style, etc.) that could help to infer the type of a player; suppose too that it is common knowledge that both know the majority language  $A$  and that only a small proportion is bilingual (there are surveys that quantify the proportion of bilingual individuals equal to  $\alpha$ ). The initial exchange of customary words and phrases, -such as, “good morning”, “how may I help you”, etc.-, will determine the language,  $A$  or  $B$ , that will be used in the subsequent conversation. Note that only when the two players happen to be bilingual (an event that occurs with probability  $\alpha^2$ ) there would be a possibility of using  $B$  in the conversation and it is only in this case that both players make a choice of language. Obviously, in the rest of the cases, that is when both players are monolingual or at least one is monolingual,  $A$  will be the language used. When only one of the players is bilingual, that player may choose  $B$ , but then he will be forced to use  $A$ .

But why is the study of the choice of language, if there is any choice at all, in a conversation important? Because the minority language  $B$  could avoid the danger of being culturally marginal or wiped out only if it is *socially used* by bilingual individuals in no matter what kind of interaction. The Language Game is a model that describes that initial stage of a conversation as a non-cooperative game in extensive form with incomplete information. The goal is to study the language conventions that the bilingual agents may build in that natural setting, since that will be an indicator of the degree of social use of language  $B$ .

The game stresses the fact that monolingual players have (trivially) perfect information during the course of the game because they do not make language choices. But the presence of monolingual players introduce uncertainties upon those who talk both languages. Bilingual players must make choices under incomplete information, because they do not know the type of player they are interacting with. Both the bilingual sender and receiver are uninformed. Given this information structure and being common knowledge that everybody speaks  $A$  and only a minority speaks  $B$ , it is not strange that in the interaction among bilingual players the majority language  $A$  is more frequently used than the minority  $B$ . Even though, and this is important, bilingual players share the common interest of using  $B$ , this is a phenomenon that happens frequently in real life situations.

A corollary of this paper, to our knowledge not stressed in sociolinguistic studies, is that since bilingual players are uninformed and since it is common knowledge that everybody speaks  $A$  and only a minority speaks  $B$ , the language contact situation produces a negative externality upon the use of the minority

language  $B$ . This effect clearly happens when only one of the players is bilingual; in that event the talk must be in the majority language  $A$ . But even when both players are bilingual they frequently fail to coordinate in the minority language  $B$ , probably because they are not informed about the bilingual nature of each other. Hence, in language contact situation, only the majority language  $A$  satisfies the main properties of a public good (*non-rivalrous consumption* –the consumption of the good by one individual does not detract from that of another–and *non-excludability*–it is difficult if not impossible to exclude an individual from enjoying the good).

The paper is organized as follows. In section 2 we present a detailed description of the game in its extensive form. We introduce two assumptions: first, that bilingual players prefer  $B$  to  $A$ , and therefore, they would get a higher (von Neumann and Morgenstern) utility when they interact in the language they prefer most than in the case when they choose to use  $A$  (see Pool (1986) and Crystal (1987)). Second, when a bilingual player, who wishes to use his preferred language, is forced to speak in his less preferred language he will suffer a frustration cost, represented by a payoff loss (see Fishman (1991)). In section 3 we carry out the equilibrium analysis of the game assuming that the frustration cost could be greater, smaller or equal to the benefit derived from using  $B$ . In each case we would have multiple Nash equilibria and in each equilibrium a specific language will be used among the bilingual players. We conclude that the most interesting case of equilibrium selection problem occurs when the cost equals the benefit.

Since speech conventions are a social construct, the equilibrium selection problem will be studied in a follow-up paper where an evolutionary setup will be used.

## 2. Description of the Language Game

Suppose a society in which there are two official languages. One of them is spoken by every individual in the society and the other by a relatively small proportion of individuals. Thus, even though, by law, they both have the same legal status, one is a majority language and the other is a minority language. To distinguish among the two official languages, let us proceed as sociolinguists do, by denoting the majority language as  $A$  and the minority language as  $B$ .

We shall assume that there are two types of individuals in the society. The bilingual type, who speaks both  $A$  and  $B$ , and the monolingual type, who only speak the majority language  $A$ . The bilingual type of individuals are a minority in this society. Let  $\alpha$  and  $(1 - \alpha)$  denote the proportion of bilingual and monolingual agents, respectively. We assume that  $\alpha < (1 - \alpha)$ , (but we should note that we



assume that  $\alpha$  is not only smaller than  $(1 - \alpha)$ , it is much smaller). Since there are sociolinguistic studies in that society, the value of  $\alpha$  is known by every individual; in other words, we shall assume that the value of  $\alpha$  is common knowledge.

Let us imagine a simple social relation in which a potential buyer enters a shop and interacts with the shopkeeper. Suppose they do not know each other and that there are no signals showing that the buyer or the seller, or both, is monolingual or bilingual. What they do know is that both speak the majority language  $A$  and that only a minority  $\alpha$  speaks  $B$ . In a society such as the one described above, bilingual players must make a decision about which language they will use in the interaction, and, very likely, that decision will be conditioned by the (speech) conventions of that society.

The actions are taken sequentially, as in an ordinary conversation. For example, a bilingual shopkeeper starts the interaction using one of the two languages and the buyer replies. If the shopkeeper chooses  $A$  and the bilingual buyer replies using  $B$ , the shopkeeper has a second chance to change or not his initial choice of language. In some cases, this decision is made out of necessity: the monolingual player dictates that the language  $A$  has to be used in the interaction. In other cases, when it happens that both agents are bilingual, they have to agree on the language they will use. In any case, as stressed above, what matters is that both the buyer and the shopkeeper do not know in advance whether the person they have to interact with is bilingual or not. This type of interaction could be represented by a non-cooperative game in extensive form with incomplete information, a Bayesian game, shown in Figure 1.

We shall assume that player I is the shopkeeper and player II the buyer. The presence of Nature represents the ignorance of each player about the other's type. Nature chooses bilingual players with probability  $\alpha$  and monolingual players  $1 - \alpha$ . Hence, in Figure 1 it is shown how Nature begins by deciding whether player I is bilingual or monolingual. After player I's move, Nature intervenes again by choosing whether player II is bilingual or not. Recall that there are studies about the knowledge of  $B$  in the society, so that we may assume that the exact values of  $\alpha$  and  $1 - \alpha$  is common knowledge among the two players.

The action set for the bilingual player is therefore  $A_{bi} = \{A, B\}$  and the corresponding set for the monolingual player is the single action set  $A_{mono} = \{A\}$ . As a consequence, when a bilingual player is matched to a monolingual player, independently of the choice of language made by the bilingual player, the interaction will necessarily take place in language  $A$ .

Players can only observe (hear) the action (language) used by the other player.

In some cases this observation reveals the type of player, but in other cases does not, as it will be seen.

A bilingual player I has two information sets:

*Bilingual Ia*: player I knows that he is a bilingual type of player, and thus in this information set he must choose the language  $A$  or  $B$  to start the conversation with player II, without knowing which type of player is II. Hence, player I is uninformed in this information set.

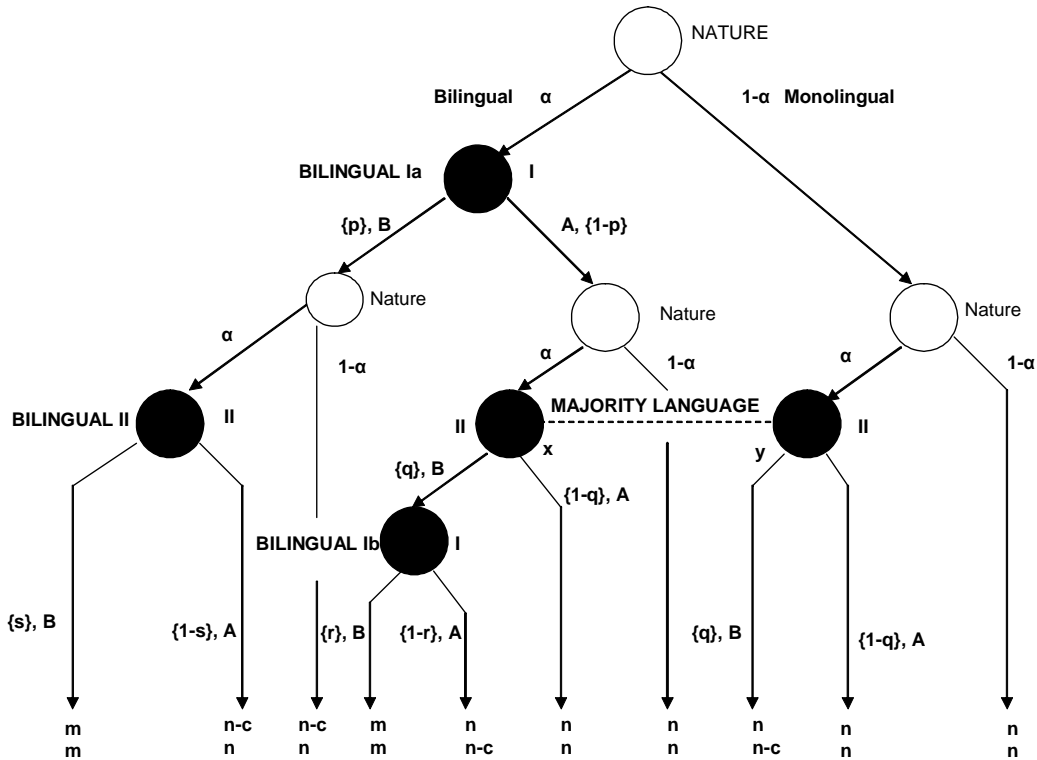
*Bilingual Ib*: this information set starts a proper subgame and player I chooses after player II has revealed that he is bilingual. Thus, player I is informed in this information set.

The bilingual player II has two information sets:

*Bilingual II*: this information set starts another proper subgame. Here, player II may choose after player I has revealed his bilingual nature because he decided at *Bilingual Ia* to choose  $B$ . Hence player II is informed in this information set.

*Majority Language (ML)*: a bilingual player II makes choices in this set after having heard player I start the conversation using the majority language  $A$ . Hence, in this set player II does not know which type of player is I, whether bilingual or monolingual. Thus, this information set contains two nodes,  $x$  and  $y$ . When player II moves in this set he must interpret the signal sent by player I and select accordingly the best outcome. For instance, if player I happens to be bilingual, both players could get the highest payoff  $m$  depending on the right choice of actions. If player I happens to be monolingual player II may get the lowest payoff  $n - c$  if he responds with  $B$ .

Monolingual players do not choose, because they can only speak  $A$ . The monolingual player I is located at the top of the game tree of figure 1 where Nature chooses the type of player I. Since the bilingual player I may start the conversation with either  $A$  or  $B$  and a monolingual one with  $A$ , there are three nodes where Nature chooses again to determine which type of player is II. Monolingual player II's action is located after each of these three nodes of Nature. Hence, in this game each bilingual player has in some information set a lack of information about the type of player he is interacting with: player I at *Bilingual Ia* and player II at *Majority Language*. After choosing an action in these two sets, both player I, the sender, and player II, the receiver, are uninformed. In the interaction they may speak  $A$  and therefore they do not reveal their type or they may reveal their nature by speaking  $B$ .



**Figure 1.** The Language Game, where A denotes the majority language, B the minority language,  $c=c(\alpha)$  the "frustration" cost and  $\alpha$  and  $1 - \alpha$  the proportion of bilingual and monolingual individuals, respectively in the society. The probability with which an action is chosen is shown by  $\{.\}$ .

## 2.1. Payoffs

The culturally specific language of any society “is more than just a tool of communication for its culture. (...). Such a language is often viewed as a very specific gift, a marker of identity and a specific responsibility *vis-à-vis* future generations.” (Fishman (1991)). This is a statement valid for any language.

Pool (1986) introduced a game with perfect information and only bilingual players,-each player’s native language is the other’s second language,- and assumed that both players prefer to speak in their own native language. Here we shall make an assumption about the preferences of the bilingual players along the same lines.

$B$  could be a bilingual player's mother tongue or not. But in any case, we shall assume that a bilingual player prefers to speak  $B$  rather than  $A$ . Formally, let  $\succ_{bi}$  denote the preference relation of a bilingual agent then, we assume

$$B \succ_{bi} A$$

Several reasons could justify that preference. Let the motivation behind that ordering be just that bilingual players are aware that  $B$  is an endangered language and that they consider that the only way to avoid its disappearance is by using it. Monolingual players do not have those concerns about the only language they speak because they know that  $A$  is not under threat. Monolingual players do not care who they are interacting with because it is common knowledge that everybody in the society speaks  $A$  and so they know that the interaction in which they are involved will take place in that language. Thus, the payoffs to a monolingual player are not affected by which language is intended to be used by the other player. Let us assume that the monolingual player gets the payoff  $n$  in the interaction. This will also be the payoff obtained by a bilingual agent who, for whatever reasons, chooses to speak  $A$  at some information set.

The efforts made by the bilingual population to reverse language shift are an indication of dissatisfaction with the cultural life which is dominated by the majority language (see Fishman (1991) p.17). Thus, one might think, that when a bilingual player, who wishes to use his preferred language, is forced to speak in his less preferred language, he will suffer a frustration that will be represented by a payoff loss of  $c(\alpha)$ . Hence, we shall make the following assumption

**Assumption 1**

For all  $\alpha \in (0, 1)$

$$m(\alpha) > n > c(\alpha) > 0$$

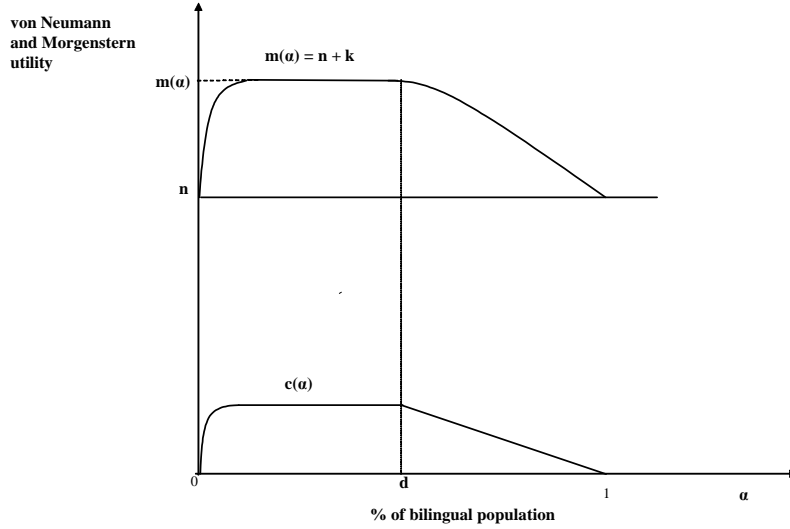
The first inequality,  $m(\alpha) > n$ , means that, for a given value of  $\alpha$ , since bilingual players prefer  $B$  to  $A$  they will get a higher von Neumann and Morgenstern utility  $m(\alpha)$  when they interact in the language they prefer most than in the case when they choose the use of  $A$ . This assumption seems to be observed by those specialized in different areas of linguistics.

“Switching to a minority language is very common as a mean of expressing solidarity with a social group. The change signals to the listener that the speaker

is from a certain background; and if the listener responds with a similar switch, a degree of rapport is established". (Crystal (1987), Chapter 60).

We may think that as the proportion of bilingual population increases and the normalization of the minority language  $B$  is being achieved, the payoff,  $m(\alpha)$ , obtained from speaking in  $B$  would get normalized too. Thus, we might assume that the von Neumann and Morgenstern utility or payoff to a bilingual player interacting in the language he prefers,  $m(\alpha)$ , is a constant function for  $\alpha \in (0, d]$ , say,  $m(\alpha) = n + k(\alpha)$  (where  $k(\alpha) = k > 0$  is a constant and  $d < 1$  should have a value of, at least,  $1/2$ ). For values of  $\alpha > d$ , that is, when the minority language  $B$  is gradually achieving a normal status, we might assume that  $m(\alpha)$  is strictly decreasing, i.e.  $\frac{\partial m(\alpha)}{\partial \alpha} < 0$ . In the limit,  $m(1) = n$  ( and in the other extreme, when no  $B$  speakers exist at all ,  $m(0) = n$  ).

The cost  $c(\alpha)$ , which we assume to be smaller than  $n$ , is intended to capture the dissatisfaction, mentioned by Fishman (1991), felt by the bilingual player who must face the fact that, in many interactions, he is forced to use the majority language. Notice that, since  $n$  is the payoff that a bilingual player can obtain for sure had he chosen to speak  $A$ , the frustration cost  $c(\alpha)$  should be subtracted from  $n$ . Thus,  $n - c(\alpha)$  is the payoff to a bilingual player who, having chosen  $B$ , is matched to someone, monolingual (or bilingual), who uses language  $A$  and, therefore, must end up speaking in  $A$ . We assume that the graph of  $c(\alpha)$  is similar to that of  $m(\alpha)$ . See Figure 2.



**Figure 2.** The payoffs

As it was mentioned above, the present work assumes a given  $\alpha$  which is much below 50% and, in figure 2, with a value in the range where the graph of  $m(\alpha)$  has reached its flat zone.

## 2.2. Pure Strategies

Player I has four pure strategies,  $S_I = \{BBA, BAA, ABA, AAA\}$ . In each strategy, from left to right, the first two components are the actions taken by a bilingual player I; the first component is the decision taken at the left information set, named *Bilingual Ia*, and the second component is the decision taken at his second information set, named *Bilingual Ib*. The last component *A* corresponds to the action of monolingual player I. Even though it is redundant, we shall keep recalling in each strategy the action of a monolingual player I. Hence, a strategy of a player describes the actions chosen when that player is bilingual and the action *A*, on the right side of each strategy, when the player is monolingual. The interpretation of player I's pure strategies is the following.

- *BBA* : If I am bilingual I will start speaking *B* and if I reach the **Bilingual Ib** information set, I would speak *B*. If I am monolingual I speak *A*. We may call this strategy “*I Speak always B*”.

- *BAA* : If I am bilingual I will start speaking *B* and if I reach the **Bilingual Ib** information set, I would speak *A*. If I am monolingual I speak *A*. This strategy could be called “*I Use B first and then switch to the language not spoken by player II*”.

- *ABA* : If I am bilingual I will start speaking *A* and if I reach the **Bilingual Ib** information set, I would speak *B* after hearing player II using *B*. If I am monolingual I speak *A*. We may call this strategy “*I Use A first and then switch to the language spoken by player II*”.

- *AAA* : If I am bilingual I will start speaking *A* and if I reach the **Bilingual Ib** information set, I would speak *A*. If I am monolingual I speak *A*. We may call this strategy “*I Speak always A*”.

Notice that Player I’s *BBA* and *BAA* strategies are equivalent because the bilingual player I’s information set **Bilingual Ib** is off the path of play and therefore he will not have the opportunity to reply when player II uses *B* at *ML* information set.

Player II’s pure strategy set is  $S_{II} = \{BBA, BAA, ABA, AAA\}$ . In each strategy, from left to right, the first two components are the actions taken when player II happens to be bilingual. The first component is the decision taken at his left information set, named **Bilingual II**; the second component is the action taken at the information set **Majority Language (ML)**, after he has heard player I speaking *A*. The third component is the action taken when he is monolingual.

- *BBA* : If I am bilingual I will always answer using *B*. If I am monolingual I speak *A*. We may call this strategy “*I Speak always B*”.

- *BAA* : If I am bilingual and I hear *B*, I will answer using *B*, but if I hear *A*, then I will answer in *A*. If I am monolingual I speak *A*. We may call this strategy “*I Speak always the language spoken by player I*”.

- *ABA* : If I am bilingual and I hear *B*, I will answer in *A*, but if I hear *A*, then I will answer in *B*. If I am monolingual I speak *A*. We may call this strategy “*I Speak always the language not spoken by player I*”.

- *AAA* : If I am bilingual and I hear *B*, I will answer speaking *A*, but if I hear *A*, then I will answer in *A*. If I am monolingual I will speak *A*. We may call this strategy “*I Speak always A* ”.

### 2.3. The Language Matrix

Each pure strategy profile can be thought of as the initial phase of a conversation between two bilingual players in which it is determined the language that could be used during the interaction. Figure 3 shows a matrix in which, given a pair of pure strategies, the corresponding entry is the language which will be used in the event that the conversation takes place between a bilingual player I and a bilingual player II (note that in each pure strategy we keep the action of a monolingual player, -the third component from left to right-, even though it has no consequences)

		<i>Player II</i>			
		<i>BBA</i>	<i>BAA</i>	<i>ABA</i>	<i>AAA</i>
Player I	<i>BBA</i>	<i>B</i>	<i>B</i>	<i>A</i>	<i>A</i>
	<i>BAA</i>	<i>B</i>	<i>B</i>	<i>A</i>	<i>A</i>
	<i>ABA</i>	<i>B</i>	<i>A</i>	<i>B</i>	<i>A</i>
	<i>AAA</i>	<i>A</i>	<i>A</i>	<i>A</i>	<i>A</i>

Figure 3. **The Language Matrix.** For each pair of pure strategies, one for each player, the corresponding entry shows the language which will be spoken when the occurring event is the matching of two bilingual players.

Four possible events or combination of player types may occur: with probability  $\alpha^2$ , both players I and II are bilingual, with probability  $\alpha(1 - \alpha)$  player I is bilingual and player II is monolingual, with probability  $(1 - \alpha)\alpha$  player I is monolingual and player II is bilingual and, finally, with probability  $(1 - \alpha)^2$  both players are monolingual. Therefore, the minority language *B* could only be spoken when the realized event is the matching of two bilingual players and if the chosen strategies by both players allow it. The Language Matrix, in Figure 3, shows the language associated to each pair of strategies when played by bilingual players (note that in this case the far right action *A* of each strategy is redundant). In the rest of the three possible events, since at least one player is monolingual, the spoken language will be *A*. For instance, let us suppose that the strategy profile



is  $(ABA, ABA)$ . In this situation, the language that will be used by the bilingual players is determined as follows: player I starts choosing  $A$  at Bilingual Ia information set and player II, after hearing language  $A$ , replies by choosing  $B$  at his Majority Language information set. Then, player I reaches his Bilingual Ib information set and there he switches to  $B$ . Hence, under this strategy profile, the minority language  $B$  will be actually spoken if the realized event is the matching of two bilingual players. The profile  $(ABA, BAA)$  shows that bilingual player I chooses  $A$  at Bilingual Ia set and then bilingual player II chooses  $A$  as well at Majority Language information set and hence, the conversation will be in language  $A$  in any of the four possible events. As a consequence, in the above matrix, 6 strategy profiles, out of 16, would allow the use of  $B$  and therefore attach the probability  $\alpha^2$  to the use of  $B$ . The other 10 strategy combinations attach zero probability to the use of  $B$ .

At first sight one would be tempted to say that a language satisfies the main properties of a public good. That is, *non-rivalrous consumption*—the consumption of the good by one individual does not detract from that of another—and *non-excludability*—it is difficult if not impossible to exclude an individual from enjoying the good. Suppose  $L$  is the official language of a society. Then, it is easy to see that if I belong to the speech community of  $L$ , I continue to enjoy the use of  $L$  at the same time that you do. By the same token, anyone in that society (i.e. the language community of  $L$ ) can enjoy the use of  $L$ . Hence, we would deduce that a language, from an economic point of view, can be thought of as a public good. But, clearly, this is not true for any language. When two languages are in contact—and one is spoken by every member of the society and the other by a minority—, the Language Game shows that even inside the speech community of the minority language  $B$  there could exist coordination failures which will block the use of  $B$ . For instance, let us consider the strategy profile  $(ABA, BAA)$ . Note that in this profile both bilingual players do wish to speak  $B$  whenever one is addressed by the other in language  $B$ . So let us assume that the event of the matching of two bilingual players is realized and that they play the above profile (excluding the  $A$  on the right of each strategy). Since player I starts the conversation using  $A$ , -but is ready to switch to  $B$  if II responds with  $B$ -, Player II is deterred from enjoying the use of  $B$  and thus he answers using  $A$ . This is because Player II does not know whether Player I is bilingual or monolingual. That is, Player II does not know in which node of the Majority Language information set is at. But, the event which would occur with a higher frequency would be the matching of monolingual players or players of different type. Then in the latter event the

conversation takes place in language  $A$  and the bilingual player would be deprived from the use of  $B$ .

Therefore, since bilingual players are uninformed and it is common knowledge that everybody speaks  $A$  and only a minority speaks  $B$ , the language contact produces a negative externality upon the use of minority language  $B$ . Hence, we should say that it is only language  $A$ , the majority language, that satisfies the main properties of a public good. This, of course, does not mean that a minority language  $B$  is a private good. We would rather say that a minority language, in a language contact situation such as the one described here, has a public good nature that suffers negative externalities from the majority language and for that reason it needs a relatively higher governmental intervention (in the form of a minority language policy) to ensure its provision and maintain diversity.

**Proposition 1**

When two languages are in contact, being one of them spoken by every member of the society (i.e. the majority language  $A$ ) and the other language is spoken by a minority group (i.e., the minority language  $B$ ), then it is language  $A$  that might be viewed as a (strict) public good. Language  $B$  has not the same status as  $A$  because, first when a bilingual player is matched with a monolingual one the property of non-excludability may not be satisfied and second, when two bilingual players are matched they may mutually exclude each other from enjoying the language  $B$  that they prefer most because they are uninformed about their true type.

**3. Equilibrium Analysis**

Note first that the Language Game has two proper subgames. One subgame starts at player I's *Bilingual Ib* information set and the other at player II's *Bilingual II* information set. In both subgames,  $B$  is the best choice for both players ( i.e.  $r = 1$  for player I and  $s = 1$  for player II) because the player who moved previously, by choosing  $B$ , has revealed that he is bilingual; thus, both players may select the outcome they prefer most.

Player I's expected payoff at *Bilingual Ia* information set is (recall that  $\alpha$  is given and, as it was mentioned above, has a small value; thus, from now on we denote  $c(\alpha)$  and  $m(\alpha)$  as  $c$  and  $m$ , respectively):

$$E_I(\text{Bilingual Ia}) = \alpha(1-p)\{q[rm + (1-r)n] + (1-q)n\} + (1-\alpha)(1-p)n + \alpha p[sm + (1-s)(n-c)] + (1-\alpha)p(n-c)$$

Therefore

$$E_I(\text{Bilingual Ia}) = \begin{cases} \alpha qr(m-n) + n & \text{if } p = 0 \\ \alpha s(m-n) + n - c(1-\alpha s) & \text{if } p = 1 \end{cases} \dots\dots\dots(1)$$

The probability of reaching player II's *Majority Language (ML)* information set is  $\alpha^2(1-p) + (1-\alpha)\alpha$ . Hence, player II's Bayes consistent beliefs at nodes  $x$  and  $y$  of *ML* information set are  $\mu_{II}(x) = \alpha(1-p)/1-\alpha p$  and  $\mu_{II}(y) = (1-\alpha)/1-\alpha p$ , respectively. Thus, the expected payoff of player II at this information set is

$$E_{II}(ML) = \frac{\alpha(1-p)}{1-\alpha p} \{q[(1-r)(n-c) + rm] + (1-q)n\} + \frac{1-\alpha}{1-\alpha p} (n-qc)$$

Therefore

$$E_{II}(ML) = \begin{cases} n & \text{if } q = 0 \\ \frac{1}{1-\alpha p} [\alpha r(m-n) - c(1-\alpha r) - \alpha pr(m-n)] & \dots\dots\dots(2) \\ -\alpha prc + \alpha p(c-n) + n & \text{if } q = 1 \end{cases}$$

Recall that player I's pure strategy set is  $S_I = \{BBA, BAA, ABA, AAA\}$ . Since *BBA* and *BAA* are equivalent, we shall refer to them as *BBA*, hence  $S_I = \{BBA, ABA, AAA\}$ , and  $\{x_1, x_2, x_3\}$  will denote the probability distribution over  $S_I$ . Let  $S_{II} = \{BBA, BAA, ABA, AAA\}$  and  $\{y_1, y_2, y_3, y_4\}$  the corresponding sets for player II.

We shall consider three cases that derive from the assumption that could be made about the possible value of the frustration cost,  $c$ , felt by bilingual players, vis-a-vis the benefit,  $(m-n)$ , this type of players might obtain when they are able to use his preferred language.

**Case 1.** Suppose that Assumption 1 is satisfied.

**Assumption 2:** For all  $0 < \alpha < 1 - \alpha$ ,

$$c = (m - n) \frac{\alpha}{(1 - \alpha)}$$

This means that, for a given  $\alpha$ , a bilingual player feels that the cost  $c$  equals the benefit  $(m - n)$  weighted by the ratio  $\frac{\alpha}{(1 - \alpha)}$ .

In societies in which the proportion of bilingual people,  $\alpha$ , is small, a bilingual person (in the role of player I), who has no information whatsoever about the person who is about to interact with, will usually start the conversation speaking language  $A$ . The reason for this convention is simple: because it is common knowledge that everybody speaks  $A$ . But the Language Game gives additional reasons. Player I, by choosing  $A$  at his *Bilingual Ia* information set, may avoid the lowest payoff  $n - c$ .

Formally, when  $r = 1$  (thus, player I chooses  $B$  in the subgame starting at *Bilingual Ib* information set) (2) becomes

$$E_{II}(ML) = \begin{cases} n & \text{if } q = 0 \\ \frac{1}{1 - \alpha p} [n - \alpha p m] & \text{if } q = 1 \end{cases}$$

If  $p = 0$  and  $r = 1$ , and this means that player I's strategy is  $ABA$ , then  $E_{II}(ML) = n$ , for all  $q$ ,  $s = [0, 1]$ . In other words, if player I starts the conversation using language  $A$  and is planning to switch to  $B$  at *Bilingual Ib*, then player II is made to be indifferent between  $A$  and  $B$  in his two information sets, which is to say that he is indifferent between all his pure strategies.

When  $s = 1$ , meaning that player II chooses  $B$  in the subgame starting at *Bilingual II* informations set, then (1) becomes

$$E_I(\text{Bilingual Ia}) = \begin{cases} \alpha q r (m - n) + n & \text{if } p = 0 \\ n & \text{if } p = 1 \end{cases}$$

We can see that if  $s = 1$  and  $q = 0$ , -that is, if player II's strategy is  $BAA$ -, then  $E_I(\text{Bilingual Ia}) = n$  for all  $p$ ,  $r = [0, 1]$ . Hence, player I is indifferent between the actions available at *Bilingual Ia* information set as well as those available at *Bilingual Ib* information set. This means that Player I is indifferent between all his pure strategies when player II chooses  $s = 1$  and  $q = 0$ .

Therefore, we have found the mixed strategy equilibrium  $(\{0, 1, 0\}, \{0, 1, 0, 0\}) = (ABA, BAA)$ , which is Bayesian perfect (here, "sequential equilibrium" and "perfect Bayesian equilibrium" are equivalent): the behaviour strategy profile  $\pi = (\pi_I, \pi_{II}) = (p = 0, r = 1; s = 1, q = 0)$  and player II's belief system  $\mu = (\mu_{II}(x) = \alpha$  and  $\mu_{II}(y) = 1 - \alpha)$  is a Bayesian perfect equilibrium for the Language Game. In this equilibrium the spoken language between bilingual players

is  $A$ .

If we keep  $p = 0$  and  $q = 0$ , then the *Bilingual Ib* and *Bilingual II* information sets are not reached with positive probability. Hence, combining the values 0 and 1 assigned to  $r$  and  $s$ , we would get Nash equilibria that are not Bayesian perfect, such as  $(ABA, AAA)$   $(AAA, BAA)$  and the pooling equilibrium  $(AAA, AAA)$ . Note that in all these equilibria, the language spoken among the bilingual players is  $A$ .

The Bayesian perfect equilibrium  $(ABA, BAA)$  might be viewed as a language coordination failure because bilingual players do intend to use the language they prefer most, but they fail and end up using  $A$ . Hence, any departure from  $p = 0$ ,  $r = 1$ ,  $q = 0$ ,  $s = 1$  that permits the realization of the desired coordination in language  $B$  would be an equilibrium too because no player would get a lower payoff and at least one player would get a higher payoff. These are the following cases:

$$(i) \text{ If } p = 1, r = 1, \text{ then } E_{II}(ML) = \begin{cases} n & \text{if } q = 0 \\ n - c & \text{if } q = 1 \end{cases}; \text{ therefore } q = 0 \text{ is}$$

the best choice and at *Bilingual II*, and since player I has revealed his bilingual nature by choosing  $p = 1$ , then player II must choose  $s = 1$ . Against  $s = 1$ ,  $q = 0$  (which means, against player II's  $BAA$ ), we have seen that player I is indifferent between any  $p$ ,  $r = [0, 1]$ ; thus  $p = 1$ ,  $r = 1$  is a best reply. Hence,  $(BBA, BAA)$  is a Bayesian perfect equilibrium (with  $\pi = (p = 1, r = 1; s = 1, q = 0)$  and  $\mu = (\mu_{II}(x) = 0 \text{ and } \mu_{II}(y) = 1)$ ), in which the spoken language among the bilingual players is  $B$ .

$$(ii) \text{ If } q = 1, s = 0, \text{ then } E_I(\textit{Bilingual Ia}) = \begin{cases} \alpha r(m - n) + n & \text{if } p = 0 \\ n - c & \text{if } p = 1 \end{cases}.$$

Hence  $p = 0$  is the best choice at *Bilingual Ia* and, since player II has revealed his type by choosing  $q = 1$ , it must be  $r = 1$  at *Bilingual Ib*. Again, if  $p = 0$  and  $r = 1$  player II is indifferent among all his pure strategies, so  $q = 1$ ,  $s = 0$  is a best choice. Thus, we get the non Bayesian perfect equilibrium  $(ABA, ABA)$  in which the spoken language among the bilingual players is  $B$ .

$$(iii) \text{ If } q = 1, s = 1, \text{ then } E_I(\textit{Bilingual Ia}) = \begin{cases} \alpha r(m - n) + n & \text{if } p = 0 \\ n & \text{if } p = 1 \end{cases}.$$

Hence, the best choices are  $p = 0$  and  $r = 1$ . Hence,  $(ABA, BBA)$  is a Bayesian perfect equilibrium (with  $\pi = (p = 0, r = 1; s = 1, q = 1)$  and  $\mu = (\mu_{II}(x) = \alpha \text{ and } \mu_{II}(y) = 1 - \alpha)$ ) in which the spoken language between bilingual players is  $B$ .

Thus, the set of equilibria for Case 1 is:

$$N = \left\{ \begin{array}{l} (ABA, BAA), (ABA, AAA), (AAA, BAA), (AAA, AAA), \\ (BBA, BAA), (ABA, ABA), (ABA, BBA) \end{array} \right\}$$

Note that player I's  $ABA$  strategy is a best response against all player II's strategies (against some strategy it is not the only one though). Hence,  $ABA$  is a weakly dominant strategy for player I. This is so because a bilingual player I who uses this strategy starts the conversation with  $A$  and with this choice he will avoid to suffer the frustration cost,  $n - c$ ; moreover, if it is matched to a bilingual player II who reveals his bilingual nature by choosing  $B$  at  $ML$  information set (i.e. a player II using either  $BBA$  or  $ABA$ ) player I then switches to  $B$  at *Bilingual Ib* information set and gets the maximum payoff  $m$ . For player II,  $BAA$  is a best response against all player I's strategies (against some strategies it is not the only one though). Thus,  $BAA$  is a weakly dominant strategy for player II, because to a bilingual player I who reveals his bilingual nature by beginning the conversation using  $B$  (when using  $BBA$  or the equivalent strategy  $BAA$ ) he will answer with  $B$  and if player I starts the conversation using  $A$  he replies with  $A$ . Therefore, the strategy profile  $ABA$  for I and  $BAA$  for II allows both players to avoid the minimum payoff  $n - c$  and reach the maximum payoff  $m$ .

We have seen that  $(ABA, BAA)$  is a Bayesian perfect equilibrium. An important feature of this equilibrium is that the linguistic convention among the bilingual players is to speak the majority language  $A$ . The equilibria in which the spoken language among the bilingual players is  $B$ , such as  $(BBA, BAA)$ ,  $(ABA, ABA)$  and  $(ABA, BBA)$  involve the play of weakly dominated strategies.

**Case 2.** Suppose that Assumption 1 is satisfied.

**Assumption 3:** For all  $0 < \alpha < 1 - \alpha$ ,

$$c > (m - n) \frac{\alpha}{(1 - \alpha)}$$

Assumption 3 says that, for a given  $\alpha$ , the cost  $c$  that a bilingual player suffers when he chooses the minority language  $B$  but is forced to switch to the majority language  $A$ , is greater than the benefit  $(m - n)$  (weighted by the ratio of bilingual population over the monolingual population,  $\frac{\alpha}{(1 - \alpha)}$ ) that he would obtain when he is able to interact using  $B$ .

Let us assume that  $p = 0$ , then from (2)

$$E_{II}(ML) = \begin{cases} n & \text{if } q = 0 \\ \alpha r(m - n) - c(1 - \alpha r) + n & \text{if } q = 1 \end{cases}$$

Hence, under assumption 3, given  $p = 0$  and any  $r \in [0, 1]$ , - which means that we are considering player I's pure strategies  $ABA$  and  $AAA$ -,  $q = 0$  maximizes  $E_{II}(ML)$ . Note that this result is independent of any  $s \in [0, 1]$ , therefore, player II will play the (strictly undominated)  $BAA$  and  $AAA$  strategies.

Let us assume now that  $q = 0$ , then from (1)

$$E_I(\text{Bilingual } Ia) = \begin{cases} n & \text{if } p = 0 \\ \alpha s(m - n) - c(1 - \alpha s) + n & \text{if } p = 1 \end{cases}$$

Therefore, given  $q = 0$  and any  $s \in [0, 1]$ , - which means that we are considering player II's pure strategies  $BAA$  and  $AAA$ -,  $p = 0$  maximizes  $E_I(\text{Bilingual } Ia)$ . This result is independent of the value of  $r \in [0, 1]$ . Therefore, player I will play the (strictly undominated)  $ABA$  and  $AAA$  strategies.

Hence, we have shown that the set of equilibria is,

$$N' = \{(ABA, BAA), (ABA, AAA), (AAA, BAA), (AAA, AAA)\}$$

where  $(ABA, BAA)$  is the only Bayesian perfect equilibrium.

Note that, as the intuition would tell us, when the frustration cost is higher than the benefits of speaking  $B$ , the language spoken among the bilingual players in all the equilibria is  $A$ .

**Case 3.** Suppose that Assumption 1 is satisfied.

**Assumption 4 :** For all  $0 < \alpha < 1 - \alpha$

$$c < (m - n) \frac{\alpha}{(1 - \alpha)}$$

Assumption 4 says that, for a given  $\alpha$ , the cost  $c$  that a bilingual player suffers when he chooses the minority language  $B$  but is forced to switch to the majority language  $A$ , is smaller than the benefit  $(m - n)$  (weighted by the ratio of bilingual population over the monolingual population,  $\frac{\alpha}{(1 - \alpha)}$ ) that he would obtain when he is able to interact using  $B$ .

Let us assume that  $p = 0$  and  $r = 1$  (in other words, assume player I's  $ABA$  strategy), then from (2)

$$E_{II}(ML) = \begin{cases} n & \text{if } q = 0 \\ \alpha(m - n) + n - c(1 - \alpha) & \text{if } q = 1 \end{cases}$$

By assumption 4,  $q = 1$  maximizes  $E_{II}(ML)$  given any  $s \in [0, 1]$  (thus against player I's *ABA*, player II's best responses are *BBA* and *BAA*).

Now suppose that  $q = 1$ , then

$$E_I(\textit{Bilingual Ia}) = \begin{cases} \alpha r(m - n) + n & \textit{if } p = 0 \\ \alpha s(m - n) - c(1 - \alpha s) + n & \textit{if } p = 1 \end{cases}$$

When  $p = 0$  and  $r = 1$  the expected payoff for player I at *Bilingual Ia* is maximized when  $q = 0$  and any  $s \in [0, 1]$ . Hence, against player II's *ABA* and *BBA* strategies, player I's best response is *ABA*. When both player I and II choose *B* at *Bilingual Ib* and *Bilingual II* information sets, the strategy profile (*ABA*, *BBA*) would be a Bayesian perfect equilibrium (that is, the behaviour strategy profile  $\pi = (\pi_I, \pi_{II}) = (p = 0, r = 1, s = 1, q = 1)$  and player II's belief system  $\mu = (\mu_{II}(x) = \alpha$  and  $\mu_{II}(y) = 1 - \alpha)$  is a Bayesian perfect equilibrium of the Language Game). (*ABA*, *ABA*) is a Nash equilibrium, but not Bayesian perfect. In both equilibria the language spoken by the bilingual players is *B*.

Suppose now that bilingual player I chooses  $p = 1$  and  $r = 1$  (that is, player I chooses *BBA*). Then,  $E_{II}(ML) = n - cq$ . Hence,  $E_{II}(ML)$  is maximized when  $q = 0$  and any  $s \in [0, 1]$ . Now, if  $q = 0$  and  $s = 1$ , then from (1) and Assumption 4 we have  $E_I(\textit{Bilingual Ia}) = \alpha(m - n) + n - c(1 - \alpha)$  if  $p = 1$  and any  $r \in [0, 1]$ . Then (*BBA*, *BAA*) is a Bayesian perfect equilibrium (where now  $\pi = (\pi_I, \pi_{II}) = (p = 1, r = 1, s = 1, q = 0)$  and  $\mu = (\mu_{II}(x) = 0$  and  $\mu_{II}(y) = 1)$ ). In this equilibrium the language spoken by the bilingual players is *B* as well.

It can be seen that (*AAA*, *AAA*) is an equilibrium too in this case, but not Bayesian perfect. Hence, the set of pure equilibria is

$$N'' = \{(\textit{ABA}, \textit{BBA}), (\textit{ABA}, \textit{ABA}), (\textit{BBA}, \textit{BAA}), (\textit{AAA}, \textit{AAA})\}$$

We can conclude that when the benefits of speaking the minority language *B* are bigger than the frustration cost, we get the obvious consequence that in all the equilibria but (*AAA*, *AAA*) the conversation among bilingual players is in *B*.

We have seen that the risk that a bilingual player faces is the loss of utility,  $c$ . The higher is this loss of utility relative to the benefit of speaking *B*,  $(m - n) \frac{\alpha}{(1 - \alpha)}$ , the higher is the risk a bilingual player would face. As a consequence, it becomes safer for a bilingual player to use the majority language *A* instead of *B*. Therefore, in equilibrium, the probability that the interaction might happen using language



$B$  decreases as the frustration cost  $c$  increases and in the limit, represented by Case 2, the minority language  $B$  is not used at all.

From the above analysis we have obtained the following result

**Proposition 2**

Given assumption 1, there exists Nash equilibria in which  $B$  could be the language used by bilingual players if, for all  $0 < \alpha < 1 - \alpha$ ,

$$c \leq (m - n) \frac{\alpha}{(1 - \alpha)}$$

$A$  will be the spoken language in equilibrium, otherwise.

**Remark**

Case II above has no interest at all because it is only  $A$  the language spoken in all the resulting equilibria and that would be the convention among the bilingual players. Case III has no theoretical interest either; we found that all the equilibria in which both bilingual players decide, at certain information set, to use  $B$  they end up speaking  $B$ . Hence, it is very likely that in the long run the convention that would be built among the bilingual players will, certainly, be to speak  $B$ .

In Case I we have a different situation. For instance, the sequential equilibrium  $(ABA, BAA)$  describes a situation in which both players intend to use  $B$  at some information set, but there is a communication failure among them and they end up speaking  $A$ . Further, this is the only equilibrium of the set  $N$  in which both players use undominated strategies. In other words, the rest of equilibria of the set of Nash equilibria  $N$  in which the spoken language is  $B$ , at least one player is using a weakly dominated strategy. Therefore, the interesting case is the situation described by Case I.

**4. Conclusion**

The Language Game highlights the incomplete information structure that bilingual players face. This may have dangerous consequences on the language diversity of the society. First, because the minority language  $B$  suffers negative externalities from the majority language  $A$ ; and second, because under some pay-off structure, it is the main reason of language  $B$  coordination failures among the

bilingual players. As a consequence, the probability that  $B$ , an already “endangered minority language”, became a “moribund language” may easily increase in the medium term. Therefore, a political intervention, in the form of language policy, is called for to ensure the provision of  $B$  and maintain the linguistic diversity of the society.

### 5. Future Work ( Part II: Dynamic Analysis)

This part studies Case I in a different setting. In real-life situations, the Language Game described previously is not an interaction that happens once. On the contrary, this game is played many times by many different people and its solution is not an individual one but a social construct. Therefore, a different analytical approach is needed. This part studies the issue of building a speech convention among the bilingual agents (which is equivalent to equilibrium selection in the set of equilibria  $N$ ) using an evolutionary approach.

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