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Ad Valorem Housing Subsidies May Reduce Housing Building

Ad valorem housing subsidies may reduce house building*

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Abstract

In this paper it is shown that an ad valorem housing subsidy set by a central regulator (or a raise in the ad valorem housing subsidy rate) may reduce the number of houses built in the market and increase the price paid by the buyers of houses. The analysis considers a situation where there is imperfect competition in the housing market and a local regulator that decides on density, or on the number of sites for housing development, and that cares about a combination of the profits of housing developers and the surplus of buyers of houses.

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1 Introduction

It is generally accepted that housing subsidies increase the number of houses built in a housing market. As housing subsidies shift the demand curve for houses to the right, the increase in the number of houses depends on the elasticity of the supply and demand curves in the housing market considered and on the level of the housing subsidy. However, in this paper it is shown that ad valorem housing subsidies may reduce house building. This is the case if housing developers compete *à la Cournot* and there is a local regulator that decides on density, or on the number of sites for housing development, and that cares about a combination of the profits of housing developers and the surplus of buyers of houses. As a consequence, the price paid by the buyers of houses may increase with ad valorem housing subsidies.[1]

The model considered in this analysis captures some relevant facts. Central governments often set housing subsidies that are ad valorem, as it occurs with tax credits and tax deductibles of housing (property and mortgage) expenses in personal income taxation. Competition in the housing market is, in general, imperfect and housing developers have market power. Local governments may have considerable control over the supply of houses. They decide on the supply of land for housing development by issuing building permits or they may regulate housing density by limiting the height of apartment buildings, establishing a minimum size for individual lots or restricting the type of housing development (single family or multi family). Furthermore, local governments frequently finance a significant proportion of their expenses through development fees for the right to build houses on privately owned land. The revenue that local authorities obtain in this way permits them to finance public services for local residents.[2]

In this paper it is noted that local authorities may prefer to limit house building in order to increase their revenues from development fees. This occurs even though local authorities care also about consumer surplus in the housing market and not only about their revenues from the development of sites. This behavior of local governments makes market failure important in the housing market, as it may enhance oligopolistic control over housing development.[3]

[1] For a discussion on the effects of housing subsidies on the housing stock see Sinai and Waldfogel (2005).

[2] On these aspects of local government behavior see Hanushek and Quigley (1990).

[3] This analysis applies to housing markets where there are not many sites available for

In this context it is considered that there is a central government that subsidizes the purchase of houses using ad valorem housing subsidies. The level of the ad valorem housing subsidy decided by the central government is taken as given in this paper. The analysis focuses in the interaction between the decisions of the local government and those of housing developers.

The analysis centers on the market for new houses, as it is assumed that substitution effects between new houses and houses built in the past are negligible. However, when there is a significant substitution effect between used and new houses, it will still be true that ad valorem housing subsidies may reduce house building and increase the price paid by the buyers of houses. Nevertheless, this result is less likely when used houses constitute an important proportion of the stock of houses.

The paper is organized as follows: Section 2 presents the housing market model. Section 3 analyzes the decision of the local government. Section 4 discusses three extensions of the previous analyses and, finally, section 5 concludes.

2 Model

Assume that the market for new houses is made up by housing developers, who decide quantities as in a Cournot oligopoly, and by many price-taking buyers. All sites for new housing development are homogeneous and each developer builds houses only on one particular site.[4] All new houses offered in the market are identical from the consumer's point of view. The unit cost of production of new houses, represented by c, is constant. There is perfect and complete information.

Let x_j be the number of new houses on site j with $j = 1, ..., n$, where n is the number of sites devoted to housing development. The inverse housing demand function for new houses is denoted by $h(x)$, where $x = \sum_{j=1}^{n} x_j$ is the total number of new houses.[5] The level of $h(x)$ depends on mortgage interest

housing development or where the opportunity cost of land used for housing development is very high for most sites. As a consequence, the number of potential housing developers is not big. The analysis fits more to mature cities than to new cities under quick expansion.

[4]Let us consider that the extension of a site is big enough to allow several-story buildings to be built on it. Hence, a site is bigger than a lot, and we may see it as closer to a subdivision.

[5]If there exist U used houses and a used house is equivalent to a proportion β of a new house with $0 < \beta \leq 1$ (the value of a used house for a buyer is a proportion β of the

rates and population in the housing market studied, but these variables are assumed to be fixed for this study. The analysis below requires the following assumption on the demand function:

Assumption A: For any x the inverse demand function $h(x)$ is such that $h(0) > c$, $h'(x) < 0$ and the marginal revenue is decreasing, i.e.,

$$2h'(x) + h''(x)x < 0.$$

Clearly, if $h(x)$ is concave or linear ($h''(x) \leq 0$), decreasing and such that $h(0) > c$, then Assumption A will be satisfied.

Each buyer of a house receives as subsidy an amount equal to sp, where p is the market price of a house and s is the ad valorem housing subsidy rate established by the central regulator, with $0 < s < 1$.[6] This subsidy is considered as given for the analysis. Hence, the game has two stages. In the first one the local regulator decides on housing density or on the number of sites for housing development. In the second stage housing developers, which compete *à la Cournot*, decide the number of houses to be built.

Let us solve the two-stage problem by backward induction in order to look for a subgame perfect equilibrium.

Second stage:

The relationship between market price p and the market sale level x will be such that

$$h(x) = (1 - s)p,$$

as $(1 - s)p$ is the net price paid by each buyer of a house.

Housing developer j, $j = 1, ..., n$, will solve the following problem:

$$\max_{x_j} \left(\frac{h(x)}{1 - s} - c\right)x_j$$

and the solution, $x_j(n, s)$ and $x(n, s) = \sum_{j=1}^{n} x_j(n, s)$, will satisfy

value of a new house for that buyer), the analysis below will remain valid using $h(x+\beta U)$, instead of $h(x)$, as the inverse housing demand function for new houses. In this case a used house might be sold at a price equal to the price of a new house multiplied by β.

[6] If there are used houses let us consider that only purchases of new houses are subsidized. Alternatively, we could consider that the purchase of a used house also receives an ad valorem housing subsidy and that the seller of a used house pays taxes equal to that subsidy.

$$h'(x(n,s))x_j(n,s) + h(x(n,s)) = c(1-s). \quad (1)$$

From (1) we have that $x_i(n,s) = x_j(n,s)$ for all $i, j \in \{1, ..., n\}$, with $i \neq j$, and, hence, the distribution of houses among sites will be symmetric. Assumption A and this symmetric distribution of houses among sites guarantee that the second order condition is satisfied. Adding conditions (1) for $j = 1, ..., n$ we obtain

$$n\left[h(x(n,s)) - c(1-s)\right] + h'(x(n,s))x(n,s) = 0. \quad (2)$$

Differentiating in (2) and using Assumption A it is not difficult to show that $x(n,s)$ increases with n and s and that $x_j(n,s)$ increases with s but decreases with n for all $n > 1$.

First stage:

The decision of the local regulator is analyzed in section 3. The goal of the local regulator is to maximize a weighted sum of the profits of housing developers and the buyers of houses surplus, given the ad valorem housing subsidy rate. He does not take into account the social cost of subsidies.

The local regulator cares about consumer surplus because buyers of houses vote in local elections. Several reasons may justify the inclusion of the profits of housing developers in the objective function of the local regulator. We can consider that the local regulator attains a proportion of the profits of the housing developers through development fees or payments in kind. That proportion would be the result of a bargaining process, not specified here, between the local regulator and the housing developers. Development fees will be the case referred in the analysis of this paper. Alternatively, we may assume that housing developers pay proportional taxes on the profits they obtain and the local government collects these taxes directly, or that the central government collects these taxes and transfers part of the revenues obtained to the local government. Another possibility would be to consider that there is (partial) capture of the local regulator by housing developers.[7]

To simplify the analysis, let us consider that the revenues obtained by the local regulator from housing developers do not affect the demand for houses in a significant way, as they finance expenses that do not make more valuable the ownership of a house.

[7] See Laffont and Tirole (1993) for a general analysis of the problem of capture of the regulator by economic agents.

3 Decision of the local regulator

In this section it is considered that the local regulator decides on housing density. The case where the local regulator decides on the number of sites for housing development will be discussed in the next section.

Assume that there are n sites where houses will be built. Let us denote by γ, with $\gamma > 0$, the relative weight of the profits of housing developers with respect to the surplus of buyers of houses in the objective function of the local regulator. The local regulator will solve the following problem:

$$\max_x \gamma(\frac{h(x)}{1-s} - c)x$$
$$+ \left(\int_0^x h(y)dy - h(x)x\right),$$

where $x = \sum_{j=1}^n x_j$.

Assuming an interior solution the first order condition of this problem implies:

$$\gamma(\frac{h(x)}{1-s} - c) + (\frac{\gamma}{1-s} - 1)h'(x)x = 0. \tag{3}$$

The second order condition may be written as

$$h'(x) + (\frac{\gamma}{1-s} - 1)\left[2h'(x) + h''(x)x\right] < 0. \tag{4}$$

From (3) we have that the interior solution requires $\gamma > 1 - s$. Assumption A and $\gamma > 1 - s$ guarantee that (4) is satisfied. We may denote the interior solution by $x^*(\gamma, s)$ as, from (3), it does not depend on n. Let us consider the symmetric case of this interior solution, $x_i^*(\gamma, s) = x_j^*(\gamma, s)$ for all $i, j \in \{1, ..., n\}$ with $i \neq j$, without loss of generality.[8]

The interior solution given by (3) will be binding if s is such that $x_j^*(\gamma, s) < x_j(n, s)$. In this case, the number of houses built on a site will be equal to the highest number of houses allowed by the local regulator on that site. If $x_j^*(\gamma, s) \geq x_j(n, s)$ the decision on housing density will not be binding and the Cournot solution $x_j(n, s)$ will be obtained.

Now we can prove:

[8] Moreover, the consideration of an assymmetric solution would require a modification of the presentation of the second stage of the problem in the previous section.

Proposition 1 *When the local regulator decides on housing density, an increase in the ad valorem housing subsidy set by the central regulator may reduce the number of houses built.*

Proof: From (3) we obtain

$$\frac{\partial x^*(\gamma,s)}{\partial s}=\frac{\frac{\gamma}{(1-s)^2}\left[h(x^*(\gamma,s))+h'(x^*(\gamma,s))x^*(\gamma,s)\right]}{-\frac{\gamma}{1-s}h'(x^*(\gamma,s))-(\frac{\gamma}{1-s}-1)\left[h'(x^*(\gamma,s))+h''(x^*(\gamma,s))x^*(\gamma,s)\right]}.$$

Hence, the sign of $\frac{\partial x^*(\gamma,s)}{\partial s}$ is equal to the sign of $h(x^*(\gamma,s))+h'(x^*(\gamma,s))x^*(\gamma,s)$ (condition (4) implies that the denominator of $\frac{\partial x^*(\gamma,s)}{\partial s}$ is positive). To complete the proof note that $h(x^*(\gamma,s))+h'(x^*(\gamma,s))x^*(\gamma,s)$, the marginal revenue at the housing density level decided by the local regulator, may be, from (3), positive, negative or equal to zero.[9] ■

An ad valorem housing subsidy (or a raise in the ad valorem housing subsidy rate) may make density reduction attractive for the local regulator, as the higher subsidy allows a greater increase in the market price for any reduction in the number of houses built. Density reduction will occur if it induces an increase in the profits of housing developers that compensates for the decrease in the surplus of buyers of houses, given the weights of these profits and surplus in the objective function of the local regulator.[10]

When there is not a binding interior solution to the problem of the local regulator the number of houses built will be $x=\sum_{j=1}^{n}x_j(n,s)$. We know from the previous section that in this case a higher ad valorem housing subsidy increases the number of houses built.

4 Extensions

Often there is a limit (a cap) in the maximum amount of housing subsidy that a buyer of a house may receive or the situation is such that the households eligible for an ad valorem housing subsidy are those households whose incomes are below some fixed income level. In this latter case the percentage of ad valorem housing subsidy received by a household may be inversely related to her income level.[11] Moreover, sometimes the context is one in which the local regulator decides on the number of sites for housing

[9] For instance, when $h(x)=e-fx$ the sign of $\frac{\partial x^*(\gamma,s)}{\partial s}$ is equal to the sign of $2\gamma c-e$.

[10] From (3) it is easy to show that $x^*(\gamma,s)$ increases with γ, as expected.

[11] See Olsen (2003).

development, instead of deciding on housing density. In this section it is explained why, in all these cases, an increase in the ad valorem housing subsidy may reduce the number of houses built and increase the net price paid by each buyer of a house.

Consider first that there is a limit in the maximum amount of housing subsidy that a buyer of a house may receive. This limit is greater than the amount of housing subsidy that a buyer of a house would have received before the increase in the ad valorem housing subsidy. In this case a large rise in the market price of houses implies a less than proportional increase in the ad valorem housing subsidy. Hence, a large rise in the market price of houses is less likely when the ad valorem housing subsidy increases. However, there may still be a reduction in the number of houses built and a rise in the price paid by each buyer of a house. We only need to note that if the maximum amount of the subsidy is L we will have

$$\frac{h(x)}{1-s} - h(x) = L \Leftrightarrow x = h^{-1}(\frac{(1-s)L}{s}).$$

Then the relationship between the market price of a house with this ad valorem housing subsidy and the market sale level x will be:

$$p = \begin{cases} h(x) + L & \text{for } x \leq h^{-1}(\frac{(1-s)L}{s}), \\ \frac{h(x)}{1-s} & \text{for } x \geq h^{-1}(\frac{(1-s)L}{s}). \end{cases}$$

Consider now that the households eligible for an ad valorem housing subsidy are those households with incomes below some fixed income level. Assume that the valuation of housing services increases with income and that the limit in the income level established to be eligible for a housing subsidy implies that only households with willingness to pay for a house below m are eligible for a housing subsidy. Moreover, consider that the ad valorem housing subsidy received by an eligible household is inversely related to her income (and, thus, to her willingness to pay for a house). Hence, the ad valorem housing subsidy rate differs among the eligible households.[12] Finally, let us assume that the housing subsidy does not modify the order of households in terms of the maximum market price that may be charged for a house to each household.

[12] This housing subsidy is a policy of the kind considered in Nichols and Zeckhauser (1982): a targeting policy. However, sometimes housing subsidies are more a policy of the type suggested by Akerlof (1978): a tagging policy, that classifies households according to characteristics over which they have no control (disability or age, for instance).

Consider, for instance, that an eligible household with willingness to pay for a house, without the subsidy, equal to $h(x)$ receives, if she buys a house, an ad valorem housing subsidy $s(x) = f(\frac{h(x)}{m})s$, with $s'(x) > 0$, $f(1) = 0$ and $f(0) = 1$. Then the relationship between the market price of a house with this ad valorem housing subsidy and the market sale level x will be:[13]

$$p = \begin{cases} h(x) & \text{for } x \leq h^{-1}(m), \\ \frac{h(x)}{1-s(x)} & \text{for } x \geq h^{-1}(m). \end{cases}$$

Note that, if the market price of houses without the subsidy is lower than m, $s(x)$ may reduce the number of houses built and increase the price paid by each buyer of a house. With housing subsidy $s(x)$, however, these effects are less likely than in the context considered in the previous section, as now the ad valorem housing subsidy rate is inversely related to the willingness to pay for a house.

Consider, finally, that the local regulator decides on the number of sites where housing development will be allowed, instead of deciding on housing density. Then he will solve:

$$\max_n \gamma(\frac{h(x(n,s))}{1-s} - c)x(n,s)$$
$$+ \left(\int_0^{x(n,s)} h(y)dy - h(x(n,s))x(n,s) \right)$$
$$s.t. \quad n \leq N,$$

where N is the maximum number of sites that may be given over to housing development and $x(n,s)$ is the solution of (2).

Assuming an interior solution, and taking into account (2), the first order condition of this problem implies:

$$\left[\gamma(-\frac{1}{n(1-s)} + \frac{1}{1-s}) - 1 \right] h'(x(n,s))x(n,s) = 0. \tag{5}$$

From (5) the number of sites developed at this interior solution will be:[14]

$$n^* = \frac{\gamma}{\gamma + s - 1}. \tag{6}$$

[13] Note that $\frac{h(x)}{1-s(x)}$ decreases with x, as it has been assumed that the order among households with respect to tha maximum market price that may be charged for a house to each household is maintained under the subsidy.

[14] If n^* is not a natural number then the number of sites developed by the local regulator will be $\lceil n^* \rceil$ or $\lceil n^* \rceil + 1$ where $\lceil n^* \rceil$ denotes the integer part of n^*.

This solution requires[15]

$$\frac{\gamma}{\gamma+s-1} \leq N \Leftrightarrow \frac{(1-s)N}{N-1} \leq \gamma.$$

At the interior solution given by (6) we have that $\frac{dn^*}{ds} < 0$. From (2), (3) and (6) we also have,

$$\frac{dx(n^*(s),s)}{ds} = \frac{\partial(x(n^*(s),s))}{\partial s} + \frac{\partial(x(n^*(s),s))}{\partial n^*(s)}\frac{dn^*(s)}{ds}$$

$$= \frac{\frac{\gamma}{(1-s)(\gamma+s-1)}\left[h(x(n^*,s)) + h'(x(n^*,s))x(n^*,s)\right]}{-\left[(n^*+1)h'(x(n^*,s)) + h''(x(n^*,s))x(n^*,s)\right]}.$$

Hence, the sign of $\frac{dx(n^*(s),s)}{ds}$ is equal to the sign of $h(x(n^*,s)) + h'(x(n^*,s))x(n^*,s)$, the marginal revenue at the number of sites for housing development decided by the local regulator (the denominator of $\frac{dx(n^*(s),s)}{ds}$ is positive by Assumption A). A higher ad valorem housing subsidy may cause a decrease in the number of houses built as it reduces the number of sites devoted to housing development (and, hence, the number of competitors).

Note that equation (3) may be written in the form

$$\frac{\gamma}{\gamma+s-1}(h(x) - c(1-s)) + h'(x)x = 0$$

and this equation is the same as condition (2) when $n = \frac{\gamma}{\gamma+s-1}$. Then, except for the implications of considering that n must be a natural number, we have that the number of houses built will coincide when the local regulator decides on housing density and when he decides on the number of sites for housing development. We also have that the results on the effects of a higher ad valorem housing subsidy will be the same under both approaches.

5 Conclusion

In this paper it has been shown that an ad valorem housing subsidy set by a central regulator (or a raise in the ad valorem housing subsidy rate) may reduce the number of houses built in the market and increase the price paid by the buyers of houses. These results have been proved considering a context where there is imperfect competition in the housing market and a

[15] When $\frac{(1-s)N}{N-1} > \gamma$ a corner solution is obtained, as the local regulator will allow housing development in all sites available (N).

local regulator that decides on density or on the number of sites for housing development and that cares about a combination of the profits of housing developers and the surplus of buyers of houses. The analysis suggests that a policy intended to help these buyers may fail if it is implemented through housing subsidies that are ad valorem, like those often included in personal income taxation.

Changes in the context considered in this paper modify the results. If the central government subsidized the purchase of houses through fixed quantity subsidies, the number of houses built would increase with the amount of the subsidy. House building would also increase with a higher ad valorem housing subsidy when the local regulator cares only about consumer surplus.

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