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Optimal Fiscal Policy in a Multisector Model with Minimum Fiscal Expenditure Requirements

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Optimal fiscal policy in a multisector model with minimum fiscal expenditure requirements*

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Abstract

This paper investigates optimal fiscal policy in a static multisector model. A Ramsey type planner chooses tax rates on each good type as well as spending levels on each good type subject to an exogenous total expenditure constraint and requirements that some minimum amount of spending be undertaken in each sector. It is shown that optimal policy does not equally spend in each sector but instead results in one of the minimum expenditure constraints binding.

JEL Classification: H23;
Keywords: Ramsey planner; Multisector model; Minimum expenditure constraint.

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1 Introduction

At least since Ramsey (1927), it is well known that the tax structure has important effects on welfare. It was here that economists first learned one of the principal findings of public finance, that efficient taxes will be highest on goods with the lowest elasticities. But Ramsey (1927) did more than contribute to our understanding of tax structure. Perhaps equally important, this paper provided what is now one of the standard approaches for formulating optimal tax problems.[1]

The original Ramsey formulation suggested that a policy planner would choose commodity taxes which would minimize welfare losses and provide sufficient revenue to the government to finance an exogenous spending program. As economists have become more interested in dynamic models, the Ramsey formulation has been adapted to dynamic settings and led to new dynamic results. Lucas and Stokey (1983) first generalized Ramsey's approach to present a dynamic stochastic model and study the structure and time consistency of optimal fiscal and monetary policy and a number of papers have further extended this theoretical framework. Chari and Kehoe (1999) review the most important contributions of this literature.[2]

Other extensions of the Ramsey optimal tax formulation have investigated the spending side of the planners budget.[3] In this paper, we also extend the Ramsey

[1] An important alternative approach was suggested by Mirrlees (1971) which has also generated a vigorous recent literature with work by Golosov, Kocherlakota and Tsyvinski (2003), Kocherlakota (2005) and Golosov and Tsyvinski (2006).

[2] A branch of special relevance within this literature has been that analyzing optimal taxation on capital. Judd (1985) and Chamley (1986) showed that long run capital tax rates should be zero. This issue has been further studied in more general theoretical setups. For instance, Chari, Christiano and Kehoe (1994) include aggregate shocks, Jones, Manuelli and Rossi (1997) consider human capital accumulation and endogenous growth and Erosa and Gervais (2002) use an overlapping generation model.

[3] Several papers have studied the properties and implications of the optimal policy when a public consumption good is assumed. Among them, Turnovsky (1996) studies the role of a tax on consumption in enhancing growth and welfare, Judd (1999) characterizes the optimal tax on capital in the long run, Gorostiaga (2003, 2005) describe the optimal policy under non-competitive labor markets, and Cassou and Lansing (2006) study the effect of tax reforms in an endogenous growth model. Other papers introduce government spending in the model as a productive input. For instance, Jones, Manuelli and Rossi (1993, 1997) study the size of optimal tax rates on capital, Fisher and Turnovsky (1998) analyze the impact of public investment on private capital accumulation, and Cassou and Lansing (1998, 1999) explore the relationship between nonoptimal fiscal policies and the observed productivity slowdown.

formulation into the spending side, but in a way not yet investigated. Here we assume that government spending has no utility or production benefits as in Ramsey (1927), but in addition to choosing taxes, we allow the planner to choose the composition of its exogenous spending program. Although it is possible to explore this structure in a dynamic model, the presentation here is done in a static context since this allows for a clean and clear understanding of the results.[4] We also found that using a simple logarithmic utility function led to even greater analytically tractable and greater intuitive clarity so most of the paper focuses on this simple case.

One might expect that a planner faced with a choice over spending composition would choose some combination of the goods, but it is shown that in fact the planner finds it attractive to focus spending on one or the other good. Minimum expenditure constraints can be added to the planners problem to insure a mix of spending on the different goods. In such a case, we show that the planner always finds it attractive to force one of the minimum expenditure constraints to bind.

Although this result is surprising, the intuition is rather simple. Because the planner gets no benefit from the goods it consumes, one can interpret the government consumption bundle as consisting of goods that are perfectly substitutable with each other. On the other hand, consumers do get utility from consuming goods. So, given that the planner must spend at least a certain exogenous amount on goods, the planner can maximize agent utility by purchasing as much of the most expensive good as possible. This results in one of the minimum expenditure constraints binding.

To present these results in a clear format, we have organized the paper as follows. Section 2 presents the formal structure of the logarithmic utility model. This simple formulation allows analytical tractability, and formal results for this model are presented in Section 3. Section 4 goes on to investigate more general utility specifications which are not analytically tractable. In this section it is shown graphically that the formal results appear to be robust to the more general utility formulations.

[4] Cassou, Gorostiaga, Gutiérrez and Hamilton (2006) investigate a dynamic model with some similarities to the one here in an environmental context.

2 The Model

In this section we describe the corporate, consumer and government sectors and then formally specify the competitive equilibrium. This structure is then used to describe the Ramsey planner's problem as a choice of policy variables so as to maximize consumer utility subject to the economy being in a competitive equilibrium.

2.1 The corporate sector

The corporate sector consists of two types of producers who manufacture different goods. We index the producers by j and distinguish the sectors by $j = a, b$ and assume there is an equal number of firms from each sector as there are consumers.[5] In these sectors, output is created through the employment of physical capital and labor according to

$$y_j = k_j^{\alpha_j} l_j^{(1-\alpha_j)} \quad \text{for} \quad j = a, b, \tag{1}$$

where $\alpha_j \in (0, 1)$ and y_j, k_j and l_j denote output, capital input and labor input in sector $j = a, b$. In this formulation, when $\alpha_a \neq \alpha_b$, capital and labor inputs have different productive characteristics across sectors. This difference in the ability to produce goods is essential to our results.

We assume that capital and labor are free to move between sectors or between firms and are allocated according to whichever sector or firm pays the highest return. This has the effect of equating returns so that

$$r = r_j = p_j \alpha_j \frac{y_j}{k_j} \quad \text{for} \quad j = a, b, \tag{2}$$

$$w = w_j = p_j (1 - \alpha_j) \frac{y_j}{l_j} \quad \text{for } j = a, b, \tag{3}$$

where r and w denote the market capital rental rate and market wage rate and r_j, w_j and p_j denote the jth sector capital rental rate, wage rate and price of output. We will use the a good as the numeraire, so $p_a = 1$.

[5] Such an assumption is common in models emphasizing competitive price taking agents and is in part justified when production exhibits constant returns to scale. This assumption allows us to focus on a representative agent running each type of firm and a reprentative consumer consuming goods, all of whom are price takers.

2.2 The consumer sector

The consumer sector consists of many identical agents who each own k units of capital and l units of labor which is provided to the corporate sector in exchange for capital and labor income. This income is then used to purchase two types of consumption goods, c_a and c_b which yield utility according to

$$U(c_a, c_b) = \ln c_a + \ln c_b. \tag{4}$$

Choices for consumption bundles are based upon a budget constraint given by

$$\sum_{j=a,b} (1+\tau_j) p_j c_j = \sum_{j=a,b} r_j k_j + \sum_{j=a,b} w_j l_j, \tag{5}$$

where τ_j is a consumption tax chosen by the government and applied to consumption of good j. This constraint shows that income received on the right side of the budget constraint is used to purchase consumption goods on the left side of the budget constraint.

2.3 The government sector

The government engages in two types of activities. First, the government purchases goods from sector j at a level denoted by $g_j \geq 0$. These purchases are assumed to be nonproductive in utility and production. Second, the government chooses a tax policy which raises revenue to finance its expenditures. The tax instruments available for this purpose consist of the consumption taxes on each of the consumption goods which were introduced above. We interpret negative values for a tax as a subsidy. We assume the government runs a balanced budget given by,

$$\sum_{j=a,b} p_j g_j = \sum_{j=a,b} \tau_j p_j c_j, \tag{6}$$

and that government spending is a proportion of the total level of output according to

$$\sum_{j=a,b} p_j g_j = \phi \sum_{j=a,b} p_j y_j, \tag{7}$$

where $\phi \geq 0$.

In addition, we assume that the government spreads its expenditures between the two sectors. This is imposed by assuming that there are minimum values for ϕ_a and ϕ_b where $\phi_j = \frac{g_j}{y_j}$ for $j = a, b$. We will denote these minimums by ϕ_a^M and ϕ_b^M and formally write the constraint as

$$\phi_j \geq \phi_j^M \quad \text{for} \quad j = a, b \quad \text{and} \quad t \geq 0. \tag{8}$$

A natural minimum value is zero, as values less than zero imply the government is able to manufacture goods without a production function. In some cases we will assume minimum values larger than zero and motivate this as the outcome of some exogenous political process that not only chooses ϕ but also ensures that this spending is spread around on all types of goods. In other words, lobbyists for each industry ensure that their sector receives some minimum level of spending. In this set up, a situation in which lobbyists for each industry are equally effective could be imposed by assuming that $\phi_a = \phi_b = \phi$.

2.4 Competitive equilibrium

There are several types of market clearance conditions. First, input market clearance requires that capital across sectors adds up to the total capital stock,

$$k = \sum_{j=a,b} k_j \tag{9}$$

and that the total time allocation adds up to the total time available,

$$l = \sum_{j=a,b} l_j. \tag{10}$$

Second, goods market clearance requires that

$$c_j + g_j = y_j \quad \text{for} \quad j = a, b. \tag{11}$$

A competitive equilibrium is defined by the following. Given a capital stock k and a labor supply l, allocations $\{k_j, l_j, c_j, y_j : j = a, b\}$, prices $\{r_j, w_j, p_j : j = a, b\}$ and government policies $\{\tau_j, g_j, \phi_j, : j = a, b\}$ and ϕ, constitute a competitive equilibrium if the following conditions are satisfied:

(i) Given prices $\{r_j, w_j, p_j : j = a, b\}$ and taxes $\{\tau_j : j = a, b\}$ the allocation $\{k_j, l_j, c_j : j = a, b\}$ maximizes the consumer objective function (4) subject the budget constraints (5), (9) and (10).

(ii) For each firm $j = a, b$, given prices r_j, w_j, p_j the allocation k_j, l_j, y_j maximizes its profits.

(iv) The government budget constraint (6), spending requirement (7) and minimum expenditure constraint (8) hold.

(v) The capital, labor and goods markets clear as given in (9), (10) and (11).[6]

2.5 The Ramsey Problem

In the original formulation by Ramsey (1927), a planner who chose taxes so as to finance an exogenous spending program and minimize deadweight losses was envisioned. Since this pioneering work, economists have extended the Ramsey formulation to include dynamic aspects as well as consider a planner that chooses government spending on various types of productive activities. Here we also consider spending elements, but with a different twist than these other studies. In particular, government spending is assumed to be non productive as in Ramsey's original work. However, here we allow the planner to choose how it wishes to allocate its spending among the various types of goods. In particular, we consider a planner who takes ϕ as given, and chooses $\{\tau_j, g_j, \phi_j, : j = a, b\}$ so as to maximize consumer utility given by (4) and subject to the requirements of the definition of a competitive equilibrium.

3 Optimal policy

First note that straightforward optimization of the consumer's problem implies

$$c_j = \frac{rk + wl}{2(1 + \tau_j)p_j} \quad \text{for} \quad j = a, b. \tag{12}$$

[6] Equations (9) and (10) appear as both consumer budget constraints and as market clearance conditions because of the representative agent set up used here.

Next note that equality of rental rates between the two production sectors implies the price of the b good is given by

$$p_b = \frac{\alpha_a}{\alpha_b}\frac{y_a}{y_b}\frac{k_b}{k_a}. \tag{13}$$

It is possible to characterize the equilibrium allocation of capital and labor in various ways, but for our purposes it will be useful to write them as functions of fiscal policy variables. Furthermore, as will be seen shortly, it is useful to write these expressions in terms of the particular fiscal variables τ_a and ϕ_a. Given τ_a, ϕ_a and ϕ, values of τ_b and ϕ_b are implied by (6) and (7). We summarize these expressions in the following Lemma.

Lemma 1: Given τ_a and ϕ_a, the equilibrium allocation of capital and labor is given by:

$$k_a = \frac{\alpha_a}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b + \alpha_a}k, \tag{14}$$

$$k_b = \frac{[2(1-\phi_a)(1+\tau_a)-1]\,\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b + \alpha_a}k, \tag{15}$$

$$l_a = \frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b) + (1-\alpha_a)}l, \tag{16}$$

and

$$l_b = \frac{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b) + (1-\alpha_a)}l. \tag{17}$$

We can now use the expressions from Lemma 1 to obtain production levels, consumption expressions and utility function values as functions of fiscal policy. We find the utility function expression useful for our purposes and summarize the behavior of this as a function of fiscal policy variables in the following proposition.

Proposition 1: For any value of τ_a, the derivative of the utility function with respect to ϕ_a is given by

$$\frac{\partial U}{\partial \phi_a} = \frac{-2\left[(1-\phi_a)(1+\tau_a)-1\right](\alpha_b-\alpha_a)^2}{(1-\phi_a)\left[[2(1-\phi_a)(1+\tau_a)-1]\,\alpha_b+\alpha_a\right]\left[[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b)+(1-\alpha_a)\right]}.$$

Proposition 1 then implies the following corollary.

Corollary 1: For any value of τ_a:

1. If $\alpha_a = \alpha_b$, then $\frac{\partial U}{\partial \phi_a} = 0$ for all ϕ_a.

2. If $\alpha_a \neq \alpha_b$, then (a) $\frac{\partial U}{\partial \phi_a} = 0$ for $\phi_a = \frac{\tau_a}{1+\tau_a}$, (b) $\frac{\partial U}{\partial \phi_a} > 0$ for $\phi_a > \frac{\tau_a}{1+\tau_a}$, and (c) $\frac{\partial U}{\partial \phi_a} < 0$ for $\phi_a < \frac{\tau_a}{1+\tau_a}$.

Because Corollary 1 holds for any value of τ_a, it will hold at the Ramsey solution as well. This means part 2 of Corollary 1 implies the following corollary.

Corollary 2: If $\alpha_a \neq \alpha_b$, then the solution to the Ramsey problem will always occur when one of the minimum expenditure constraints is binding.

To understand Corollary 2 intuitively, first note that one can substitute (14) and (15) into (13) and then make use of two production expressions from the appendix given by (24) and (25) to get

$$p_b = \frac{\alpha_a^{\alpha_a}(1-\alpha_a)^{(1-\alpha_a)}}{\alpha_b^{\alpha_b}(1-\alpha_b)^{(1-\alpha_b)}} \times \left(\frac{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\frac{k}{l}\right)^{\alpha_a-\alpha_b}. \tag{18}$$

This expression shows that when $[2(1-\phi_a)(1+\tau_a)-1] = 1$ (i.e. $\phi_a = \frac{\tau_a}{1+\tau_a}$), the price equals the price that would be achieved in a no government situation (i.e. $\phi = \phi_a = \phi_b = \tau_a = \tau_b = 0$). Because of this, we will interpret $[2(1-\phi_a)(1+\tau_a)-1] = 1$ as a baseline for the following discussion. Note, that as ϕ_a rises (falls), $[2(1-\phi_a)(1+\tau_a)-1]$ falls (rises). So we can approach the comparative statics from either perspective and each will be used where it has an advantage.

Next note that

$$\frac{\partial p_b}{\partial \phi_a} = \frac{\alpha_a^{\alpha_a}(1-\alpha_a)^{(1-\alpha_a)}}{\alpha_b^{\alpha_b}(1-\alpha_b)^{(1-\alpha_b)}}\left(\frac{-2(1+\tau_a)(\alpha_b-\alpha_a)^2}{([2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a)^2}\frac{k}{l}\right) \times \left(\frac{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\frac{k}{l}\right)^{\alpha_a-\alpha_b-1}$$

and that this derivative is always negative.[7] This implies that as ϕ_a rises from the baseline, the price of the b good falls, implying the b good is relatively cheaper than the baseline, while when ϕ_a falls from the baseline, the price of the b good rises, implying that the a good is relatively cheaper.

Recognizing this is really all that is needed for understanding the result. Essentially the government is like a consumer who has perfectly substitutable preferences between the a good and the b good. What they want to do is provide households with the most valued consumption bundle. They can do this by using their own spending budget up on the most costly good. Thus for ϕ_a larger than the baseline, the a good is relatively costly and spending on it leaves a more preferred consumption bundle for consumers. Similarly, when ϕ_a is smaller than the baseline, the b good is relatively more costly than its baseline price and the government can leave a more preferred bundle for consumers by purchasing more of the b good.

An alternative way to see this is to work directly with implications from the Ramsey planner. First note that consumer optimization implies

$$\frac{\partial U}{\partial c_b} = \frac{p_b(1+\tau_b)}{(1+\tau_a)}\frac{\partial U}{\partial c_a}. \tag{19}$$

Since the Ramsey planner takes consumer optimization as given, this expression also holds for the Ramsey planner. Next note that Corollary 1 says that for ϕ_a larger than the baseline (i.e. $\phi_a > \frac{\tau_a}{1+\tau_a}$)

$$\frac{\partial U}{\partial \phi_a} = \frac{\partial U}{\partial c_a}\frac{\partial c_a}{\partial \phi_a} + \frac{\partial U}{\partial c_b}\frac{\partial c_b}{\partial \phi_a} > 0.^8 \tag{20}$$

Plugging in (19) and cancelling terms implies

$$\frac{\partial c_a}{\partial \phi_a} > \frac{-p_b(1+\tau_b)}{(1+\tau_a)}\frac{\partial c_b}{\partial \phi_a}.$$

[7] Alternatively note that

$$\begin{aligned}\frac{\partial p_b}{\partial[2(1-\phi_a)(1+\tau_a)-1]} \quad = \quad & \frac{\alpha_a^{\alpha_a}(1-\alpha_a)^{(1-\alpha_a)}}{\alpha_b^{\alpha_b}(1-\alpha_b)^{(1-\alpha_b)}}\left(\frac{(\alpha_b-\alpha_a)^2}{([2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a)^2}\frac{k}{l}\right)\times \\ & \left(\frac{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\frac{k}{l}\right)^{\alpha_a-\alpha_b-1}.\end{aligned}$$

is always positive.

[8] An analogous argument holds for the $\phi_a < \frac{\tau_a}{1+\tau_a}$ case and is left to the reader.

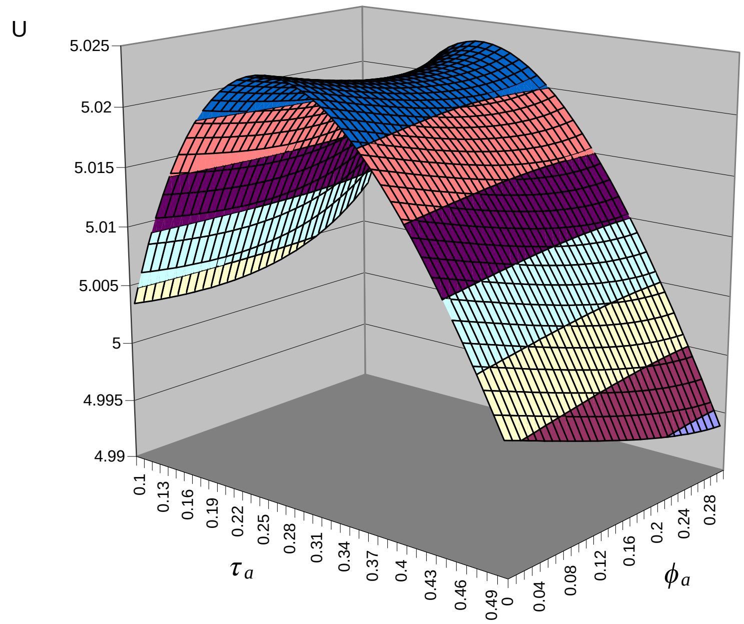
Notice that the right hand side can be interpreted as the number of a goods that can be obtained in the market for the b good reduction arising from the ϕ_a change. The equation implies that a small increase in ϕ_a implies a b good reduction and an a good increase which is such that the increase in a consumption is larger than would be obtained through simple market trading. Again what is happening is that as ϕ_a increases, the government gives up some b good which it trades back to the market to buy more of the a good. However, because the a good is relatively costly, it cannot buy very much, thus leaving more a good for consumers.[9]

Finally, an alternative way to visualize Corollary 2 is to consider Figure 1 which plots utility values implied by the competitive equilibrium for alternative values of τ_a and ϕ_a when $\alpha_a = 0.65$, $\alpha_b = 0.4$, $\phi = 0.2$, $k = 35$ and $l = 25$. If one holds τ_a constant, the cross section of Figure 1 shows a U shaped curve. This is to be expected given the results of Corollary 1, part 2. What Corollary 2 concludes is that because the cross section for any τ_a is U shaped, the Ramsey solution will be a boundary value which is clear from Figure 1.[10]

[9] Other insights can be obtained by noting how the allocation of labor and capital given by (14), (15), (16) and (17) change as one moves away from the baseline. These exercises are left to the reader.

[10] Note that if one uses minimum expenditure values of $\phi_a^M = \phi_b^M = 0$, then the optimal policy occurs at $\tau_a = .2632$ and $\phi_a = 0$ while other values for ϕ_a^M and ϕ_b^M would be located at other points in Figure 1.

Figure 1: Log utility



4 Extensions to more general utility

When extending the model beyond the logarithmic utility, simple analytical results for the Ramsey problem become infeasible. This arises because the expressions characterizing elements of the competitive equilibrium are highly nonlinear and introduce marginal conditions into the Ramsey planner's first order conditions which do not readily simplify. However, despite this limitation, it is still possible to investigate extensions numerically by using the same routines that generated Figure 1.

We considered two extensions to the model. The first is for the consumption arguments in utility to exhibit elasticities with each other that differ from the unit elasticity implied by logarithmic preferences and the second is for labor to exhibit less than perfectly inelastic behavior. We formulated these extensions with a general CES utility function given by

$$U(c_a, c_b, l) = \left(\varepsilon_1 c^\theta + (1-\varepsilon_1)(1-l)^\theta\right)^{\frac{1}{\theta}}, \tag{21}$$

where

$$c = (\varepsilon_2 c_a^{\psi} + (1-\varepsilon_2) c_b^{\psi})^{\frac{1}{\psi}}, \tag{22}$$

and parameters are restricted according to $0 < \varepsilon_1 < 1$, $0 < \varepsilon_2 < 1$, $\theta \leq 1$ and $\psi \leq 1$. The parameter θ is related to the elasticity of substitution between the consumption aggregate c and leisure and ψ is related to the elasticity of substitution between the consumption levels c_a and c_b. In this formulation, larger values of ϕ and ψ indicate greater rates of substitution while smaller values indicate greater levels of complementarity.

Our investigation carried out a grid search procedure to find the optimal policy values under the more general utility function. It was found that Corollary 1 did not generalize completely, but was true in a local region near the optimum, while Corollary 2, because it focused only on the optimal values, did generalize completely. In particular, the part of Corollary 1 which claims that a U shaped curve will hold for any value of τ_a was not always true for some elasticity values, but for τ_a near the optimum the U shaped behavior was always present. Since our interest is in finding results about Ramsey type decisions, the generalization of Corollary 2 is what is most important and leads us to conclude that the optimal policy will always occur at a boundary where one or the other minimum expenditure constraint is binding.

Figures 2 and 3 provide a sample of the types of diagrams we found. These are part of the investigation for the CES consumption good extension. Analogous diagrams for the investigation of the elastic labor extension are not included here because of their similarity to the inelastic labor case.[11] Figures 2 and 3 are drawn for $\psi = .5$ and $\psi = -1.5$ respectively while $\varepsilon_2 = 0.5$ and the other model parameters are the same as those used in Figure 1. These curves are drawn so that attention can be focused on the optimal policy outcome.[12] The diagrams clearly show that for policy values near the optimum, holding τ_a constant, produces a U shaped curve along the ϕ_a dimension. This shows that Corollary 1 does generalize in a local region near the

[11] These diagrams can be obtained from the authors upon request.

[12] Assuming minimum expenditure values of $\phi_a^M = \phi_b^M = 0$, the optimum values occur at $\tau_a = .245$ and $\phi_a = 0$ for the model in Figure 2 and $\tau_a = .555$, $\phi_a = .335$ and $\phi_b = 0$ for the model in Figure 3.

optimum. Furthermore, because of this U shaped behavior for local values of τ_a, we see that Corollary 2 generalizes completely since it focuses on the optimum.

Figure 2: CES utility with $\psi = .5$

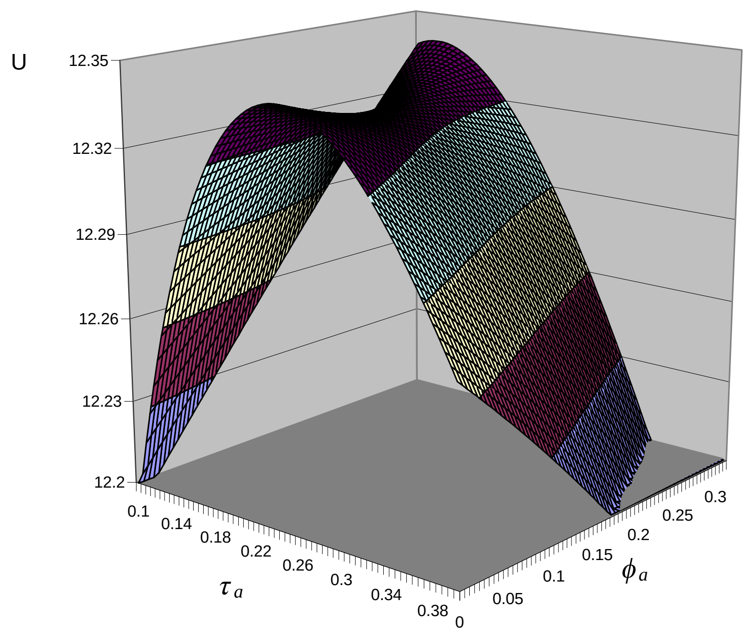
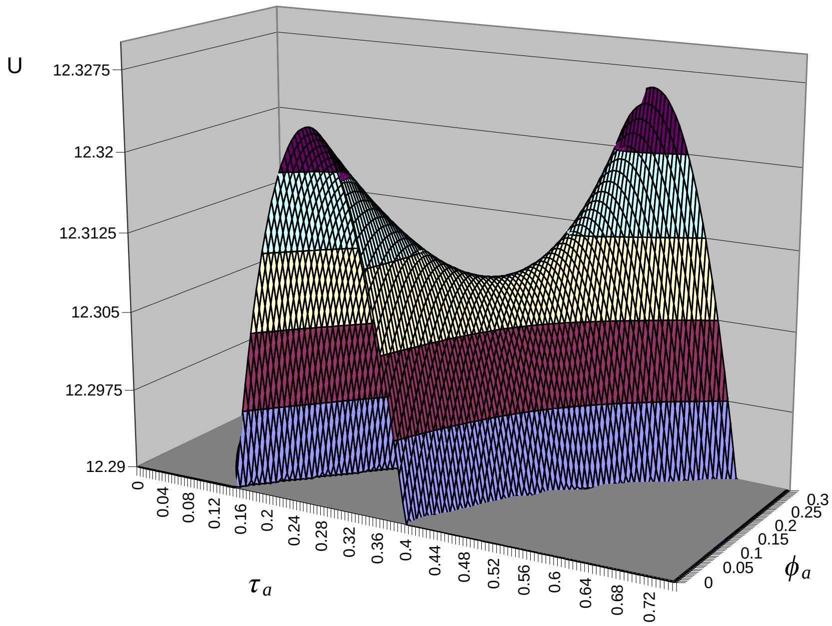


Figure 3: CES utility with $\psi = -1.5$



5 Conclusion

This paper investigated an extension to the Ramsey planning structure in which the planner is able to choose the composition of its spending. Using a simple two good static model with logarithmic preferences it was shown that such a planner chooses boundary values for its own consumption. It was also shown that these boundary solutions appear to be robust to more general utility formulations. In a related dynamic model, Cassou, Gorostiaga, Gutiérrez and Hamilton (2006) also find these boundary solutions. An intuitive interpretation for this result is offered which suggests a planner finds it attractive to allocate its sending toward the most costly good. This intuitive interpretation supports the finding that the result occurs in analytically intractable settings with more general utility formulations or dynamic environments.

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A Appendix

This appendix goes through the proof of various propositions in the paper.

A.1 Proof of Lemma 1

Because the production technologies exhibit constant returns to scale technology, income equals GDP and can be written as

$$rk + wl = y_a + p_b y_b.$$

The market clearing condition for the A good market implies that: $c_a = y_a - g_a = (1-\phi_a)y_a$, which upon making use of (12) gives

$$c_a = \frac{y_a + p_b y_b}{2(1+\tau_a)} = (1-\phi_a)y_a.$$

Substituting in (13) gives

$$y_a + \frac{\alpha_a}{\alpha_b}\frac{y_a}{y_b}\frac{k_b}{k_a}y_b = 2(1-\phi_a)(1+\tau_a)y_a,$$

which can be written as

$$\alpha_b k_a + \alpha_a k_b = 2(1-\phi_a)(1+\tau_a)\alpha_b k_a. \tag{23}$$

Using the capital market clearance condition (9), the amount of capital in sectors a and b can be solved to get

$$k_a = \frac{\alpha_a}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b + \alpha_a}k,$$

and

$$k_b = \frac{[2(1-\phi_a)(1+\tau_a)-1]\,\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b + \alpha_a}k.$$

Next using (3), one gets

$$(1-\alpha_a)\frac{y_a}{l_a} = (1-\alpha_b)\,p_b\frac{y_b}{l_b}.$$

Using (13), one gets an expression for capital and labor ratios of

$$\frac{(1-\alpha_a)}{(1-\alpha_b)}\frac{k_a}{k_b} = \frac{\alpha_a}{\alpha_b}\frac{l_a}{l_b}.$$

Using this jointly with the equilibrium capital allocation, one can get

$$\frac{l_a}{l_b} = \frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b)}.$$

Finally, using (10) one can get

$$l_a = \frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b) + (1-\alpha_a)}l$$

and

$$l_b = \frac{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b) + (1-\alpha_a)}l.$$

A.2 Proof of Proposition 1

Plugging (14), (15), (16) and (17) into (1) and rearranging gives

$$\begin{aligned} y_a &= \left(\frac{\alpha_a}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_a} \times \\ & \left(\frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(1-\alpha_a)} \Omega_a \qquad (24) \\ y_b &= [2(1-\phi_a)(1+\tau_a)-1]\left(\frac{\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_b} \times \\ & \left(\frac{(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(1-\alpha_b)} \Omega_b \\ &= [2(1-\phi_a)(1+\tau_a)-1]\, y_b'. \qquad (25) \end{aligned}$$

where $\Omega_j = k^{\alpha_j} l^{(1-\alpha_j)}$ for $j = a, b$, and

$y_b' = \left(\frac{\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_b}\left(\frac{(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(1-\alpha_b)}\Omega_b.$

The market clearing condition for the j good implies that $c_j = y_j - g_j = (1-\phi_j)y_j$ for $j = a, b$. Using this along with (25) in (4) gives

$$U(c_a, c_b) = \ln(1-\phi_a) + \ln(1-\phi_b) + \ln[2(1-\phi_a)(1+\tau_a)-1] + \ln y_a + \ln y_b'.$$

Differentiating the utility function with respect to ϕ_a gives

$$\frac{\partial U}{\partial \phi_a} = -\frac{1}{(1-\phi_a)} - \frac{1}{(1-\phi_b)}\frac{\partial \phi_b}{\partial \phi_a} - \frac{2(1+\tau_a)}{2(1-\phi_a)(1+\tau_a)-1} + \frac{1}{y_a}\frac{\partial y_a}{\partial \phi_a} + \frac{1}{y_b'}\frac{\partial y_b'}{\partial \phi_a}. \qquad (26)$$

We now evaluate each of the terms in this derivative beginning with the second term. Begin by using (13) and $g_j = \phi_j y_j$ for $j = a, b$ in (7) to get

$$\phi_b = \phi - \frac{\alpha_b}{\alpha_a}\frac{k_a}{k_b}(\phi_a - \phi).$$

Next, note that this implies

$$\begin{aligned} 1-\phi_b &= 1-\phi + \frac{1}{2(1-\phi_a)(1+\tau_a)-1}\phi_a - \frac{1}{2(1-\phi_a)(1+\tau_a)-1}\phi \qquad (27) \\ &= \frac{2(1-\phi)(1+\tau_a)-1}{2(1-\phi_a)(1+\tau_a)-1}(1-\phi_a). \end{aligned}$$

Also, using (14) and (15) we get $\frac{k_a}{k_b} = \frac{\frac{\alpha_a}{\alpha_b}}{2(1-\phi_a)(1+\tau_a)-1}$, which implies

$$\phi_b = \phi - \frac{1}{2(1-\phi_a)(1+\tau_a)-1}(\phi_a - \phi).$$

We now compute the derivative of ϕ_b with respect to ϕ_a to get

$$\begin{aligned}\frac{\partial\phi_b}{\partial\phi_a} &= -\frac{2(1-\phi_a)(1+\tau_a)-1-(\phi_a-\phi)(-2)(1+\tau_a)}{[2(1-\phi_a)(1+\tau_a)-1]^2} \\ &= -\frac{2(1-\phi)(1+\tau_a)-1}{[2(1-\phi_a)(1+\tau_a)-1]^2},\end{aligned}$$

and using (27) we see

$$\frac{\partial\phi_b}{\partial\phi_a} = -\frac{(1-\phi_b)}{(1-\phi_a)}\frac{1}{[2(1-\phi_a)(1+\tau_a)-1]}.$$

Next use (24) to get

$$\begin{aligned}\frac{\partial y_a}{\partial\phi_a} &= \alpha_a\left(\frac{\alpha_a}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_a-1}\left(\frac{\alpha_a 2(1+\tau_a)\alpha_b}{([2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a)^2}\right)\times \\ &\quad \left(\frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(1-\alpha_a)}\Omega_a \\ &\quad +(1-\alpha_a)\left(\frac{\alpha_a}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_a}\times \\ &\quad \left(\frac{(1-\alpha_a)2(1+\tau_a)(1-\alpha_b)}{([2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a))^2}\right)\times \\ &\quad \left(\frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(-\alpha_a)}\Omega_a,\end{aligned}$$

which implies

$$\frac{1}{y_a}\frac{\partial y_a}{\partial\phi_a} = \left(\frac{2(1+\tau_a)\alpha_a\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)+\left(\frac{2(1+\tau_a)(1-\alpha_a)(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right).$$

Similarly, note that (25) gives

$$\begin{aligned}\frac{\partial y_b'}{\partial\phi_a} &= \alpha_b\left(\frac{\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_b-1}\left(\frac{\alpha_b 2(1+\tau_a)\alpha_b}{([2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a)^2}\right)\times \\ &\quad \left(\frac{(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(1-\alpha_b)}\Omega_b \\ &\quad +(1-\alpha_b)\left(\frac{\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_b}\times \\ &\quad \left(\frac{(1-\alpha_b)2(1+\tau_a)(1-\alpha_b)}{([2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a))^2}\right)\times \\ &\quad \left(\frac{(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(-\alpha_b)}\Omega_b\end{aligned}$$

which implies

$$\frac{1}{y_b'}\frac{\partial y_b'}{\partial\phi_a} = \left(\frac{2(1+\tau_a)\alpha_b^2}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)+\left(\frac{2(1+\tau_a)(1-\alpha_b)^2}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)$$

Substituting out these derivatives in (26) gives

$$\begin{aligned}\frac{\partial U}{\partial \phi_a} &= -\frac{1}{(1-\phi_a)} - \frac{1}{(1-\phi_b)}\left(-\frac{(1-\phi_b)}{(1-\phi_a)}\frac{1}{[2(1-\phi_a)(1+\tau_a)-1]}\right) - \frac{2(1+\tau_a)}{2(1-\phi_a)(1+\tau_a)-1} \\ &\quad + \left(\frac{2(1+\tau_a)\alpha_a\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right) + \left(\frac{2(1+\tau_a)(1-\alpha_a)(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right) \\ &\quad + \left(\frac{2(1+\tau_a)\alpha_b^2}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right) + \left(\frac{2(1+\tau_a)(1-\alpha_b)^2}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right).\end{aligned}$$

Putting things over a common denominator and rearranging gives

$$\frac{\partial U}{\partial \phi_a} = \frac{-2\left[(1-\phi_a)(1+\tau_a)-1\right](\alpha_b-\alpha_a)^2}{(1-\phi_a)\left[\left[2(1-\phi_a)(1+\tau_a)-1\right]\alpha_b+\alpha_a\right]\left[\left[2(1-\phi_a)(1+\tau_a)-1\right](1-\alpha_b)+(1-\alpha_a)\right]}.$$